Gaussian Elimination Meets Maximum Satisfiability

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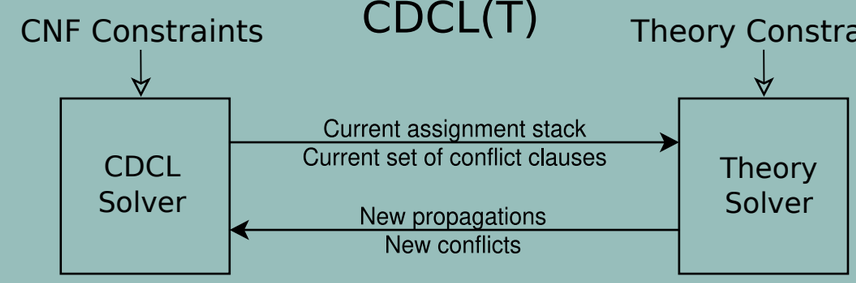
MaxSAT

CNF but with
* Hard constraints, cannot be violated
* Soft constraints, can be violated with cost attached
* Goal: solve hard constraints, minimize cost violation of soft constraints

MaxHS is a highly performing MaxSAT solver [Davies and Bacchus 2013]

CDCL(T)



Provides search algorithm:
* New variables set
* Backtracking

Provides theory solver:
* New propagations and conflicts
* Reason for prop/confl

Gauss-Jordan Elimination as a Theory to CDCL

We need to keep two matrices, one assigned, and one unassigned to store reason

$\mathbf{S}-matrix$ with x_3 assigned to `true`

x_1	x_2	x_3	x_4	x_5	aug
1	1	0	1	1	0
0	**1**	**0**	**0**	**0**	**0**
0	0	0	1	1	1
0	**0**	**0**	**0**	**1**	**0**
0	0	0	0	0	0

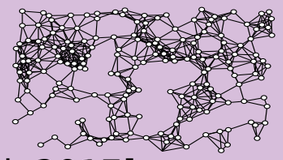
$\mathbf{R}-matrix$

x_1	x_2	x_3	x_4	x_5	aug
1	1	0	1	1	0
0	**1**	**1**	**0**	**0**	**1**
0	0	0	1	1	1
0	**0**	**1**	**0**	**1**	**1**
0	0	1	0	0	1

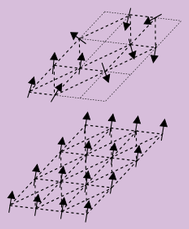
S-matrix row 2 indicates propagation of x2
R-matrix row 2 indicates reason

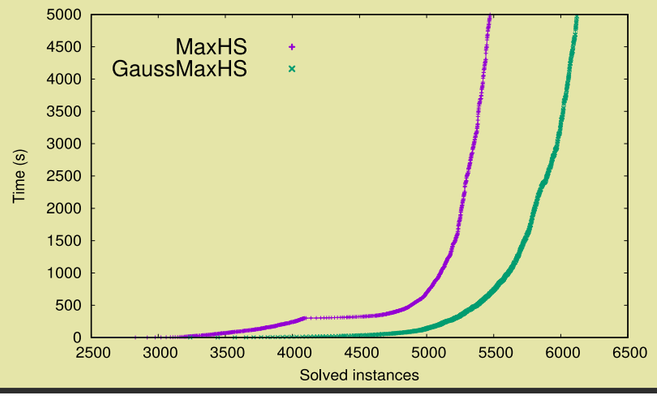
Challenge Instances: Spin Glass, Network Reliability

Network Reliability
Power grid modeled as a (V,E) graph
Compute probability of 2 points connected
Reduced to discrete integration [Duenas-Osorio et al. 2017]



Spin Glass
Magnetic state characterized by randomness
Variables represent spins {-1, +1}
Used 7x7 spin glass model as per [Kuck et al. 2018]





Evaluation Results

Instance	vars	cls	xors	Max HS	Gauss MaxHS	Speed-up
sping2-x_11	49	193	11	246.70	15.67	15.74
sping1-x_6	49	193	9	22.43	1.77	12.64
sping2-x_5	49	193	50	0.02	0.02	1.00
sping3-x_11	49	193	22	1557.77	116.89	13.33
sping5-x_3	49	193	15	310.97	9.70	32.06
sping5-x_15	49	193	29	-	2930.45	-
sping5-x_11	49	193	45	-	0.01	-
sping6-x_7	49	193	46	-	0.02	-

Instance	vars	cls	xors	Max HS	Gauss MaxHS	Speed-up
Net12_24_13_cnt_106	191	320	14	591.33	10.84	54.57
Net12_24_13_cnt_106	191	320	12	596.24	21.08	28.29
Net22_60_8_cnt_116	219	350	22	-	353.91	-
Net22_60_8_cnt_116	219	350	9	40.54	8.99	4.51
Net3_51_19_cnt_65	117	197	21	2652.90	671.09	3.95
Net22_84_16_cnt_116	219	350	190	-	501.74	-
Net22_84_16_cnt_116	219	350	196	-	16.22	-
Net27_81_58_cnt_118	230	356	202	-	1133.91	-
Net27_81_58_cnt_118	230	356	199	-	4825.98	-
Net27_90_62_cnt_118	230	356	202	-	311.05	-

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