# Designing New Phase selection Heuristics

Arijit Shaw • Kuldeep S. Meel

# Conflict Driven Clause Learning (CDCL)

**CDCL(F)**

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**Decision Heuristics**

**Clause Maintainance Heuristics**

**Restart Heuristics**

# Development of Solvers

**New heuristics or, betterment of existing ones**

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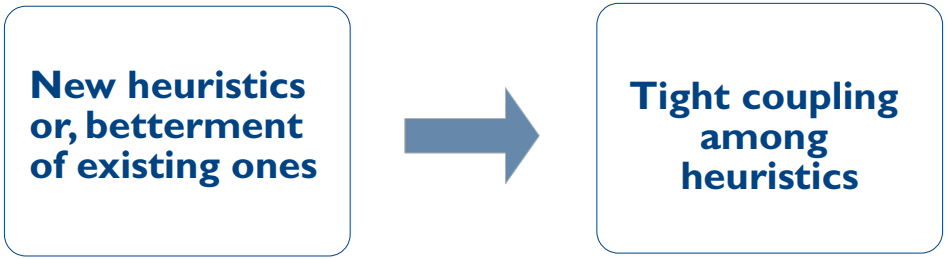

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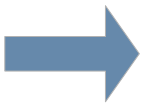

**Tight coupling among heuristics**

unclear whether VMTF only works in combination with Glucose restarts

**Evaluating CDCL Variable Scoring Schemes (SAT'15)**

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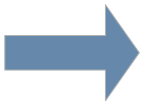

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**Between SAT and UNSAT: The Fundamental Difference in CDCL SAT (SAT'15)**

Chanseok Oh

we decided to use the (VSIDS decay) factor of 0.999 in the no restart phase. For the Glucose (restart) phase, however, we retained the default value of 0.95

# Heuristics under study

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## Who uses?


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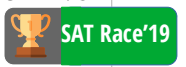


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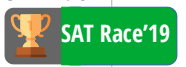


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**A Lightweight Component Caching Scheme For Satisfiability Solvers (SAT'07)**

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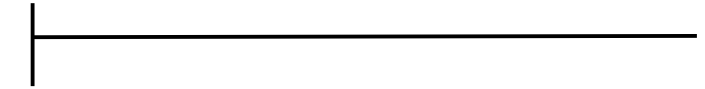


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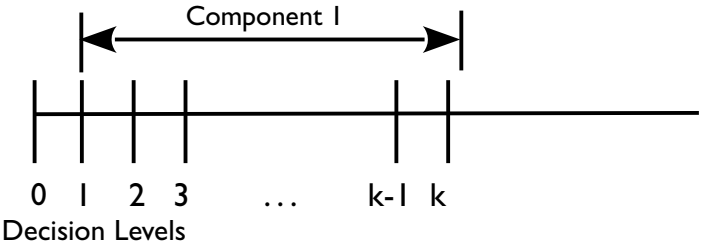

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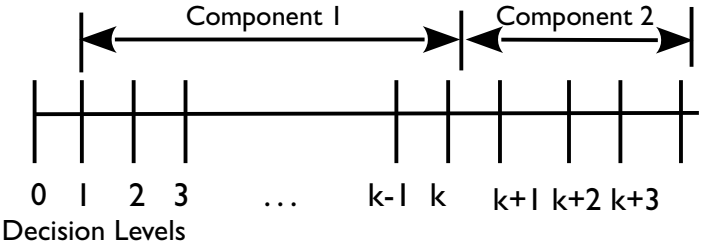

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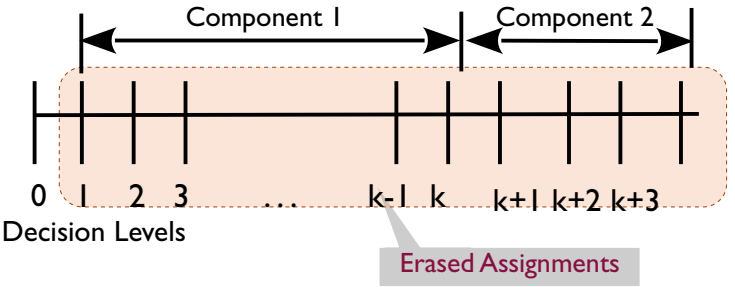

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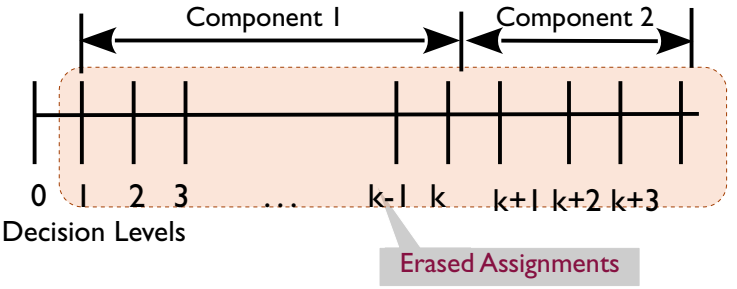


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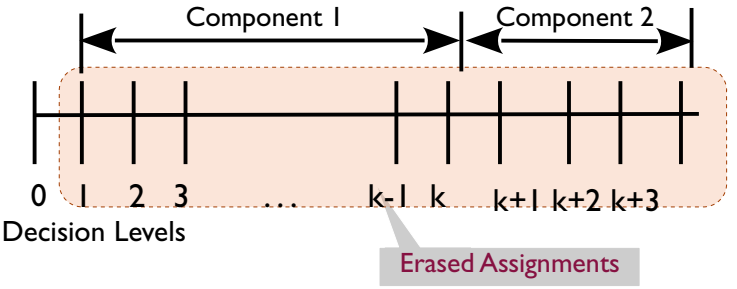


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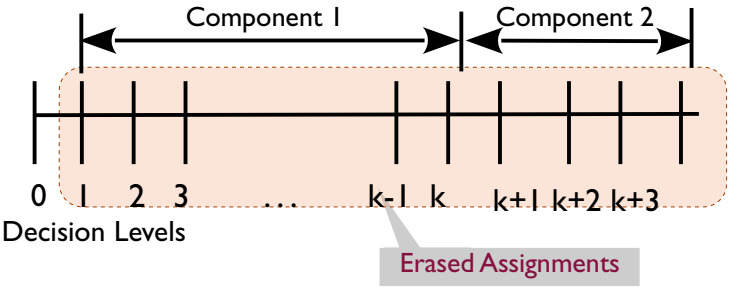


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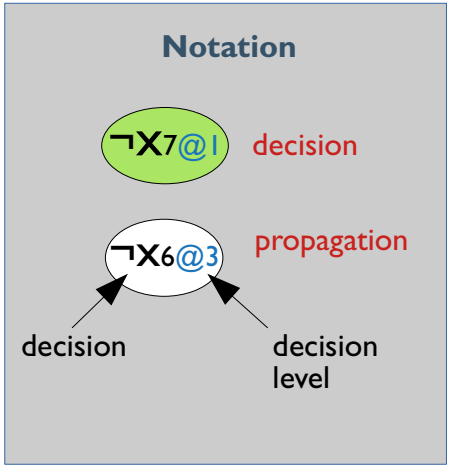

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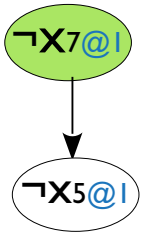


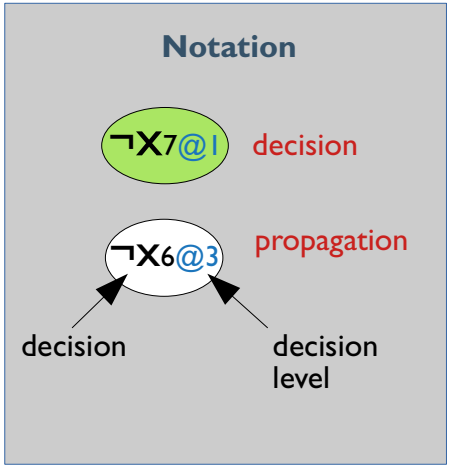

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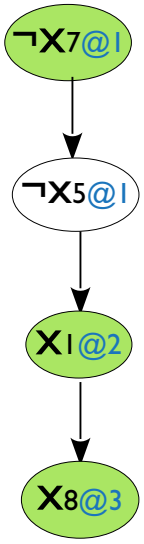


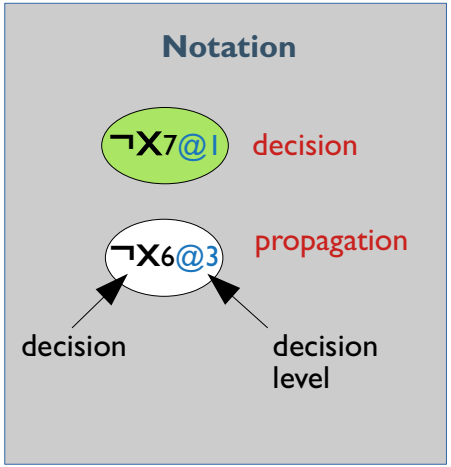

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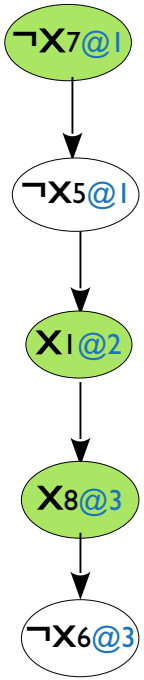


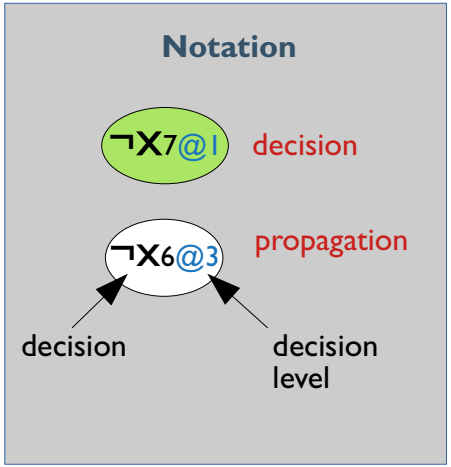

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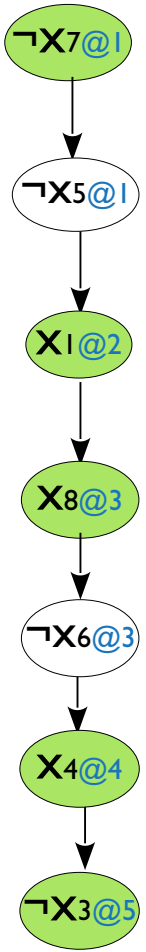


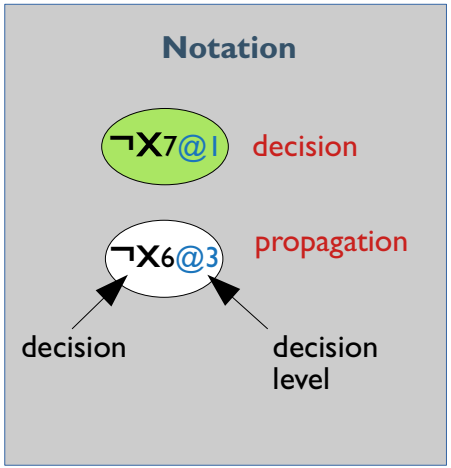

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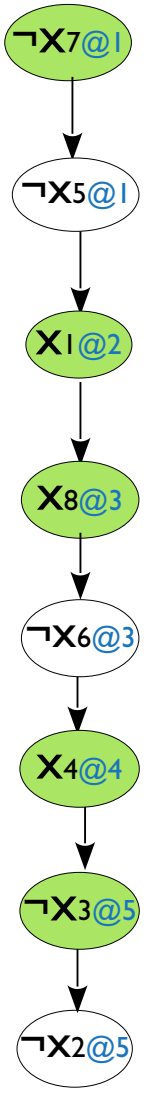


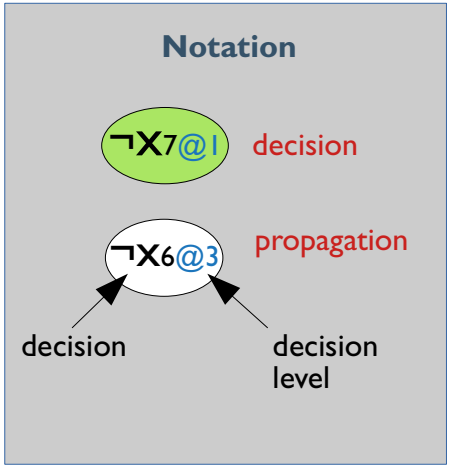

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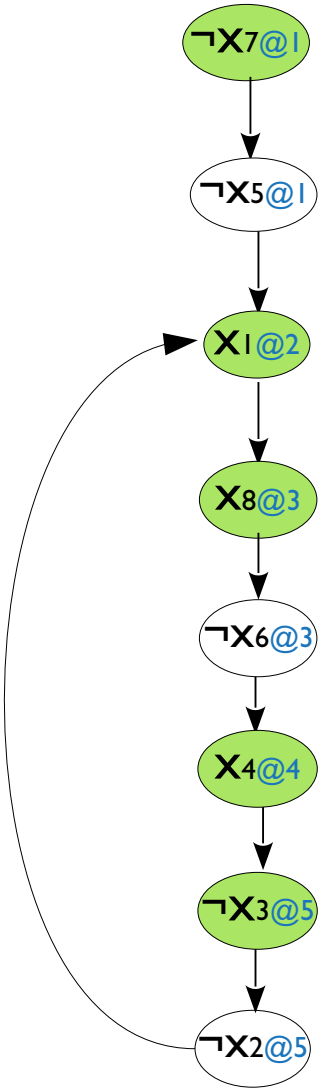


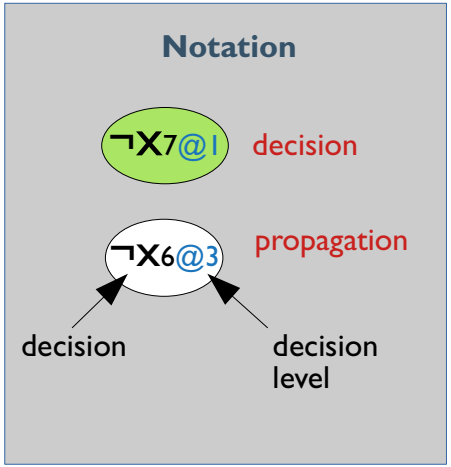

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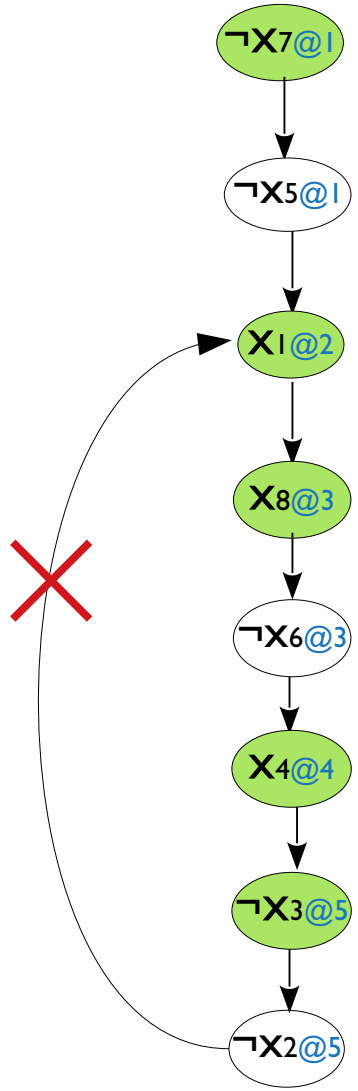


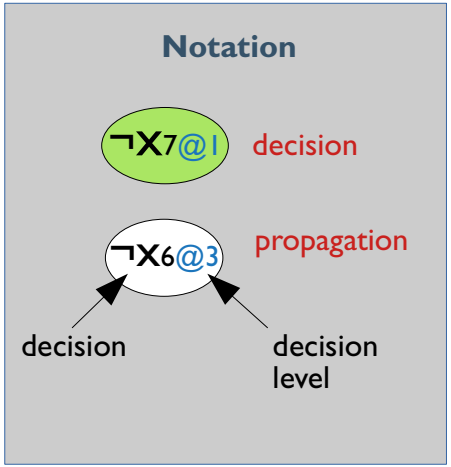

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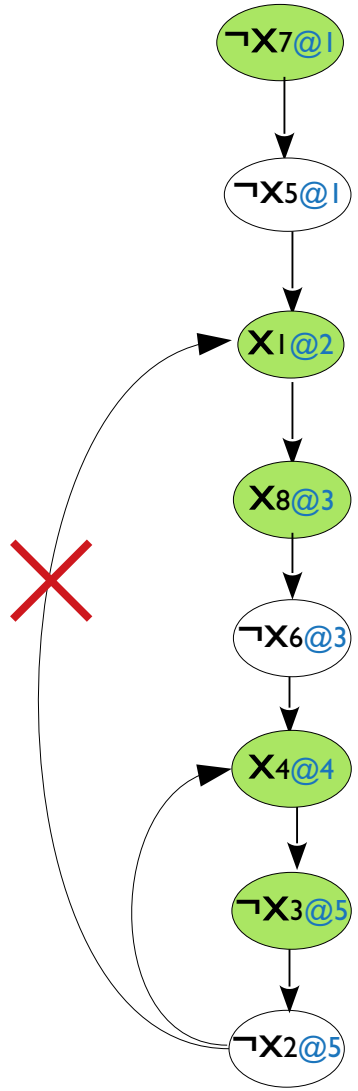


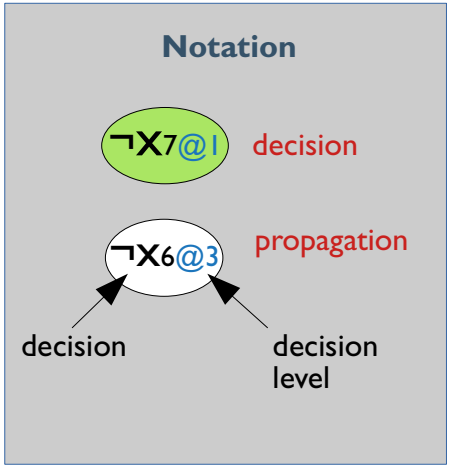

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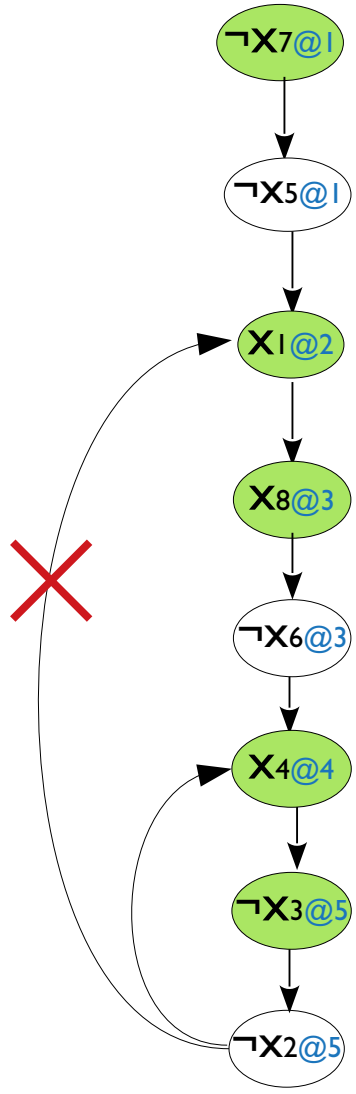


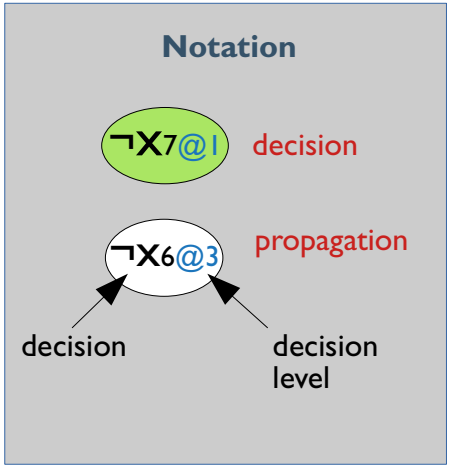

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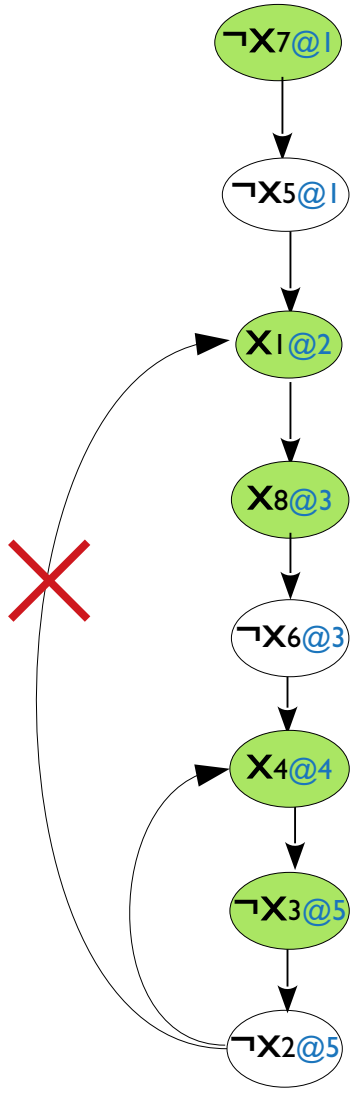


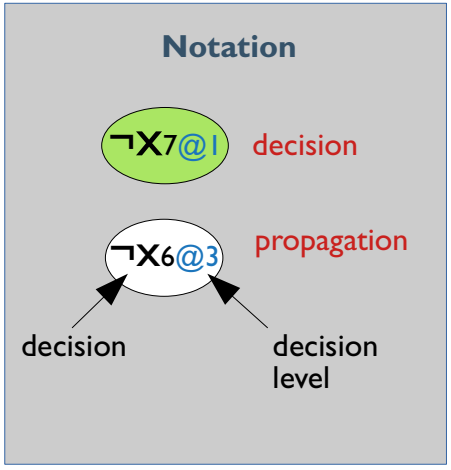

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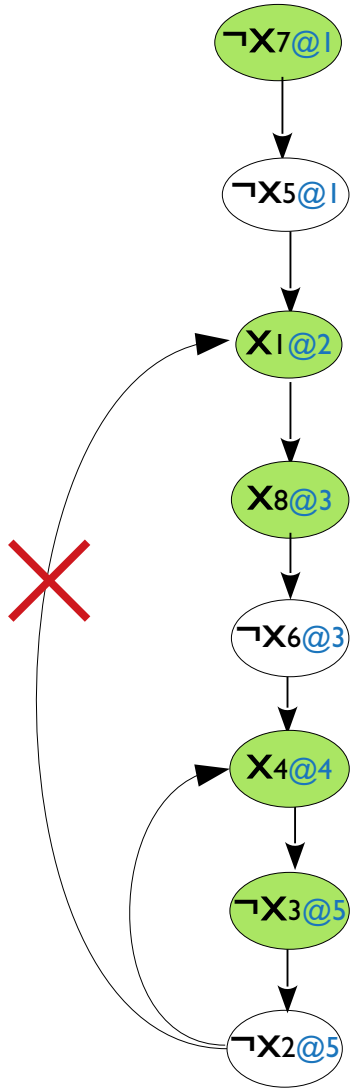


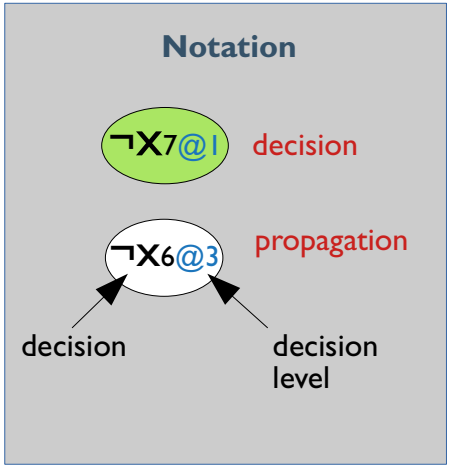

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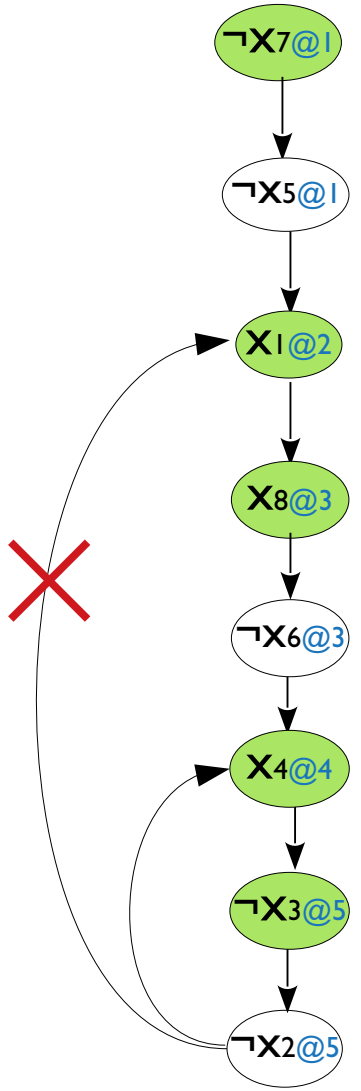
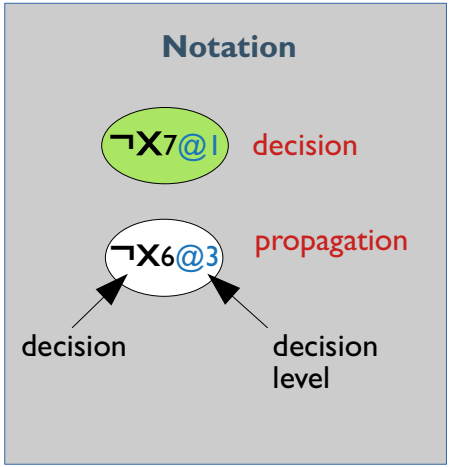
Question : Is phase saving still useful, if the solver backtracks chronologically?



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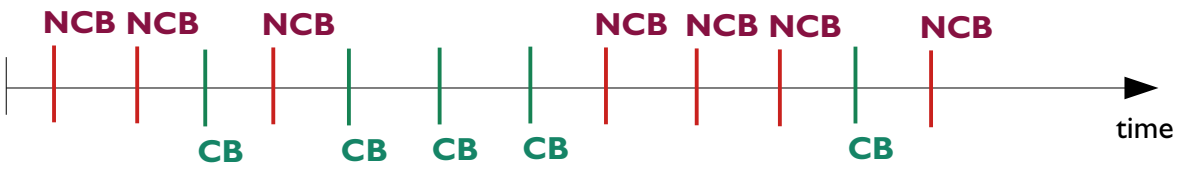

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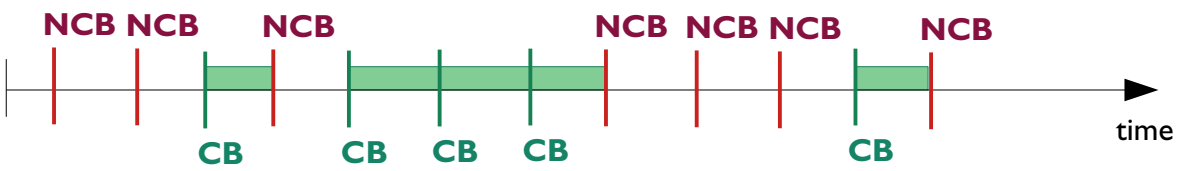

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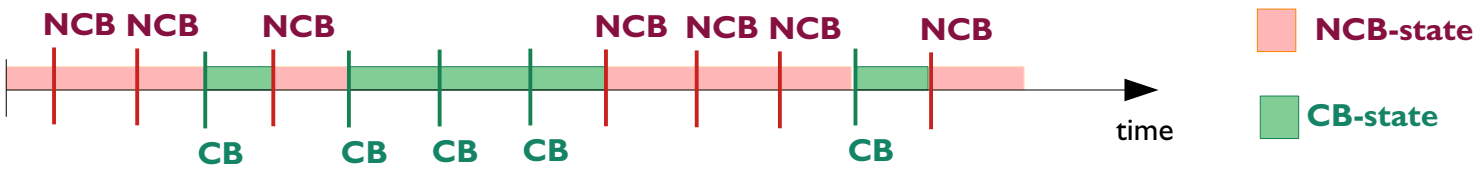

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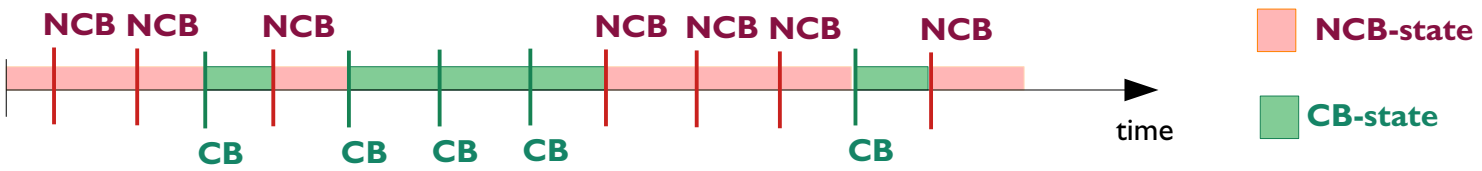


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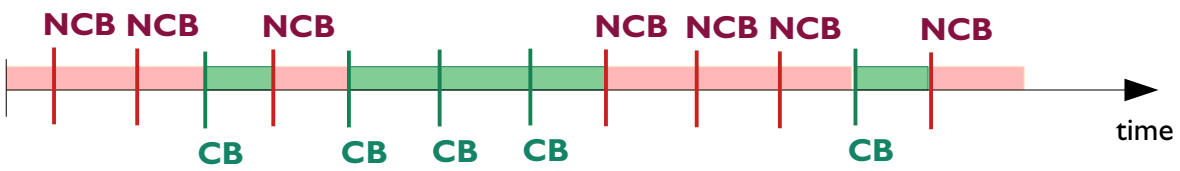


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| Phase Selection Heuristic used | | Instances solved | Avg. Runtime* |
|---|---|---|---|
| NCB-state | CB-state | | |
| Phase Saving | Phase Saving | | |
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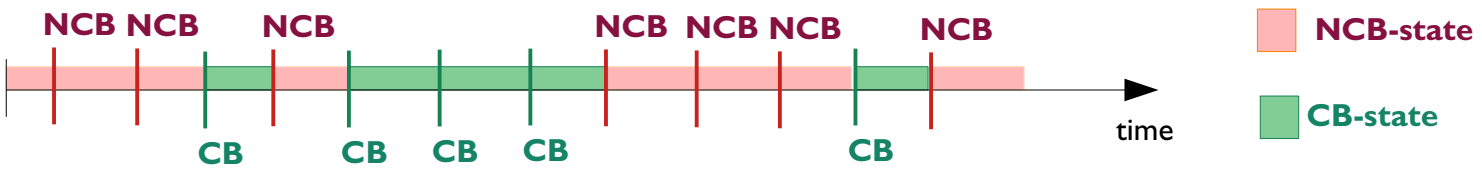


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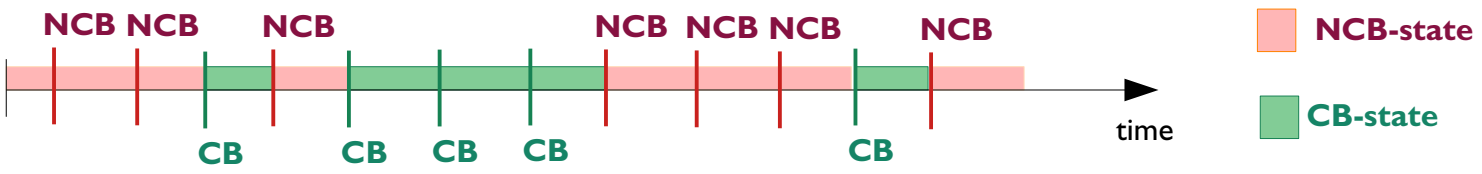


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* **Base Solver :** Maple_LCM_Dist_ChronoBT_v3
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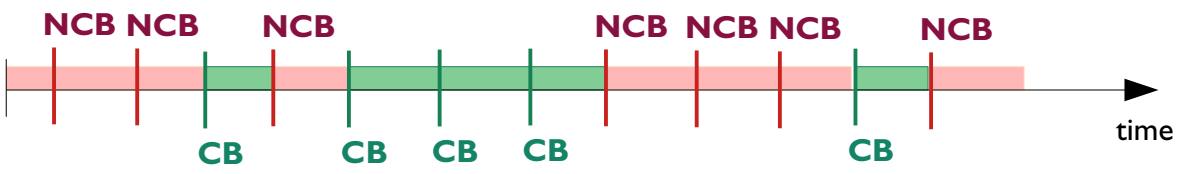


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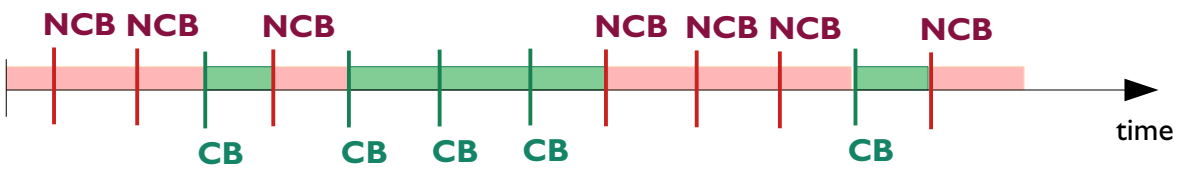


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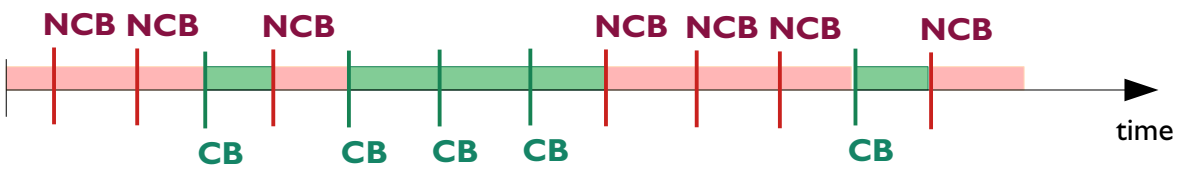


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## Phase Saving

| x1 | x2 | x3 | x4 | x5 |
|---|---|---|---|---|
| ⊥ | ⊥ | ⊤ | ⊥ | ⊤ |

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**return** SavedPhase(v)

Data Strucure

Update during backtrack

Phase Selection

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## DPS

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|---|---|---|---|---|
| 2.50 | - 0.5 | - 9.3 | 7.9 | 0.25 |

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| Update during backtrack | **SavedPhase**(v) = assignment(v) | **DPS**(v) = λ·DPS(v) + polarity(v)<br>0.5 < λ < 1.0<br>polarity( ⊤ ) = +1<br>polarity( ⊥ ) = - 1 |
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on SAT '19 instances

| solver | # solved | Avg. Runtime* |
|---|---|---|
| MLDC | 237 | 4556 |
| MLDC_DPS | 239 | 4585 |

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Maintain the essence of **phase saving**

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**Example!**

**Learnt Clauses :**

c1 : ¬x1 ∨ x2 ∨ ¬x4

c2 : ¬x1 ∨ ¬x2 ∨ x3

c3 : x3 ∨ ¬x4

| x1 | ¬x1 | x2 | ¬x2 | x3 | ¬x3 | x4 | ¬x4 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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| | x1 | ¬x1 | x2 | ¬x2 | x3 | ¬x3 | x4 | ¬x4 |
|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Bump** | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |

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|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Bump** | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
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c3 : x3 ∨ ¬x4

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|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Bump** | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| **Decay** | 0 | 0.8 | 0.8 | 0 | 0 | 0 | 0 | 0.8 |
| **Bump** | 0 | 1.8 | 0.8 | 1 | 1 | 0 | 0 | 0.8 |

# LSIDS : a scoring scheme for literals

## Literal State Independent Decaying Sum

**Example!**

**Learnt Clauses :**

c1 : ¬x1 ∨ x2 ∨ ¬x4

c2 : ¬x1 ∨ ¬x2 ∨ x3

c3 : x3 ∨ ¬x4

| | x1 | ¬x1 | x2 | ¬x2 | x3 | ¬x3 | x4 | ¬x4 |
|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Bump** | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| **Decay** | 0 | 0.8 | 0.8 | 0 | 0 | 0 | 0 | 0.8 |
| **Bump** | 0 | 1.8 | 0.8 | 1 | 1 | 0 | 0 | 0.8 |
| **Decay** | 0 | 1.44 | 0.64 | 0.8 | 0.8 | 0 | 0 | 0.64 |

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|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Bump | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| Decay | 0 | 0.8 | 0.8 | 0 | 0 | 0 | 0 | 0.8 |
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c3 : x3 ∨ ¬x4

**Backtrack :**

| | x1 | ¬x1 | x2 | ¬x2 | x3 | ¬x3 | x4 | ¬x4 |
|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Bump** | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| **Decay** | 0 | 0.8 | 0.8 | 0 | 0 | 0 | 0 | 0.8 |
| **Bump** | 0 | 1.8 | 0.8 | 1 | 1 | 0 | 0 | 0.8 |
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**Backtrack :**

¬x2

| | x1 | ¬x1 | x2 | ¬x2 | x3 | ¬x3 | x4 | ¬x4 |
|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Bump** | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| **Decay** | 0 | 0.8 | 0.8 | 0 | 0 | 0 | 0 | 0.8 |
| **Bump** | 0 | 1.8 | 0.8 | 1 | 1 | 0 | 0 | 0.8 |
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**Backtrack :**

¬x2

| | x1 | ¬x1 | x2 | ¬x2 | x3 | ¬x3 | x4 | ¬x4 |
|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Bump** | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| **Decay** | 0 | 0.8 | 0.8 | 0 | 0 | 0 | 0 | 0.8 |
| **Bump** | 0 | 1.8 | 0.8 | 1 | 1 | 0 | 0 | 0.8 |
| **Decay** | 0 | 1.44 | 0.64 | 0.8 | 0.8 | 0 | 0 | 0.64 |
| **Bump** | 0 | 1.44 | 0.64 | 0.8 | 1.8 | 0 | 0 | 1.64 |
| **Bump** | 0 | 1.44 | 0.64 | 2.8 | 1.8 | 0 | 0 | 1.64 |

# LSIDS : a scoring scheme for literals

## Literal State Independent Decaying Sum

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**Backtrack :**

¬x2

| | x1 | ¬x1 | x2 | ¬x2 | x3 | ¬x3 | x4 | ¬x4 |
|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Bump** | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| **Decay** | 0 | 0.8 | 0.8 | 0 | 0 | 0 | 0 | 0.8 |
| **Bump** | 0 | 1.8 | 0.8 | 1 | 1 | 0 | 0 | 0.8 |
| **Decay** | 0 | 1.44 | 0.64 | 0.8 | 0.8 | 0 | 0 | 0.64 |
| **Bump** | 0 | 1.44 | 0.64 | 0.8 | 1.8 | 0 | 0 | 1.64 |
| **Bump** | 0 | 1.44 | 0.64 | 2.8 | 1.8 | 0 | 0 | 1.64 |

## Phase Selection

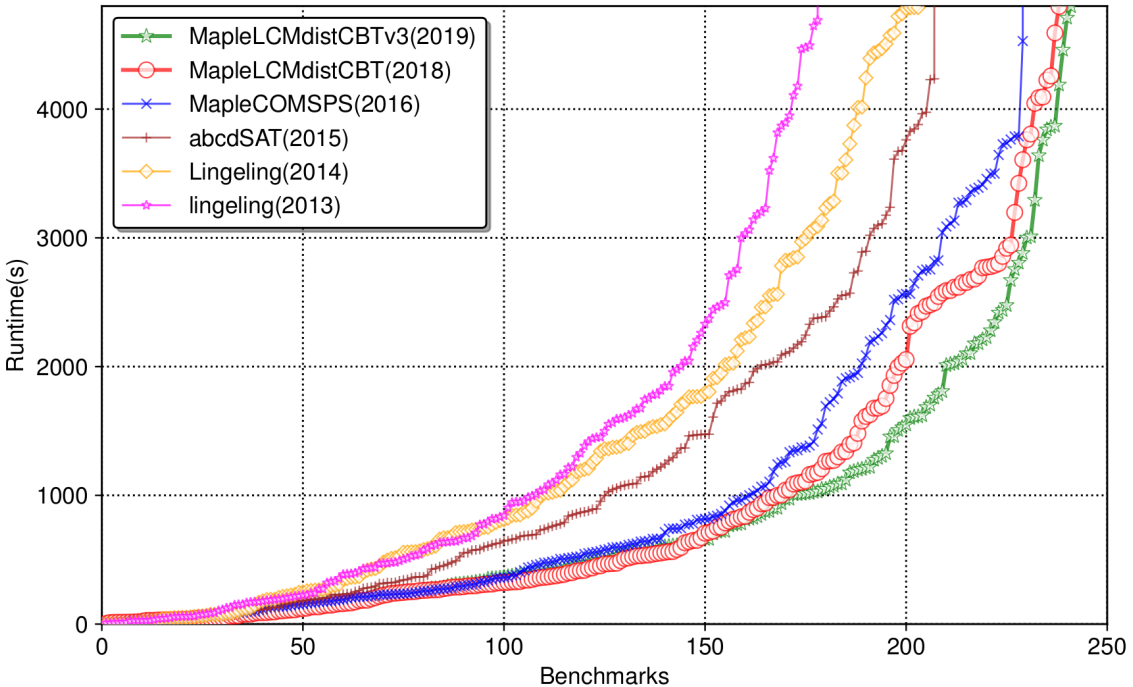
- Compare score of two literals of the variable.
- Choose the one with higher score.

# SAT Revolution

## over years 2013 - 19

# SAT Revolution

## over years 2013 - 19




on SAT '19 benchmarks
5000s timeout

| solver | year | # solved |
|---|---|---|
| lingeling | 2013 | 179 |
| lingeling | 2014 | 188 |
| abcdSAT | 2015 | 202 |
| MapleCOMSPS | 2016 | 224 |
| MapleLCMDistCBT | 2018 | 233 |
| MapleLCMDistCBTv3 | 2019 | 237 |

# Results

on 400 SAT '19 instances

| solver | # solved | Avg. Runtime* |
|---|---|---|
| MLDC | | |
| MLDC_LSIDS | | |

*** MLDC :** Maple_LCM_Dist_ChronoBT_v3

***Avg. Runtime :** PAR-2 scores

# Results

on 400 SAT '19 instances

| solver | # solved | Avg. Runtime* |
|---|---|---|
| MLDC | 237 | 4556 |
| MLDC_LSIDS | **243** | **4398** |

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**Room for being skeptical**

- Is LSIDS complete noise?

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on 500 cryptographic instances

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|---|---|---|
| MLDC | | |
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| MLDC | 291 | 9939 |
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**T :** CB if (decision level - backtracking level) > T
**C :** NCB for first C conflicts

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|---|---|---|
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*Avg. Runtime : PAR-2 scores


**T :** CB if (decision level - backtracking level) > T
**C :** NCB for first C conflicts

| | | T = 100 | | | | C = 4000 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | C = 2000 | C = 3000 | C = 4000 | C = 5000 | T = 25 | T = 90 | T = 150 | T = 200 |
| **# solved** | MLDC | 235 | 237 | 235 | 234 | 237 | 233 | 229 | 235 |
| | MLDC -LSIDS | **242** | **240** | **243** | **239** | **241** | **238** | **238** | **239** |
| **Avg. Run-time** | MLDC | 4663 | 4588 | 4556 | 4674 | 4609 | 4706 | 4773 | 4641 |
| | MLDC -LSIDS | **4506** | **4558** | **4398** | **4575** | **4555** | **4556** | **4622** | **4583** |

# Key Contribution

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