A Scalable and Nearly Uniform Generator of SAT Witnesses

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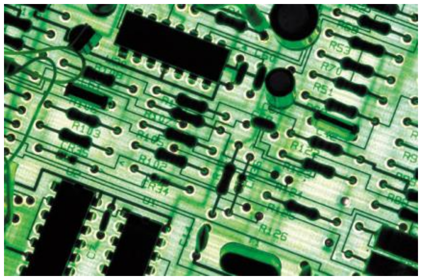
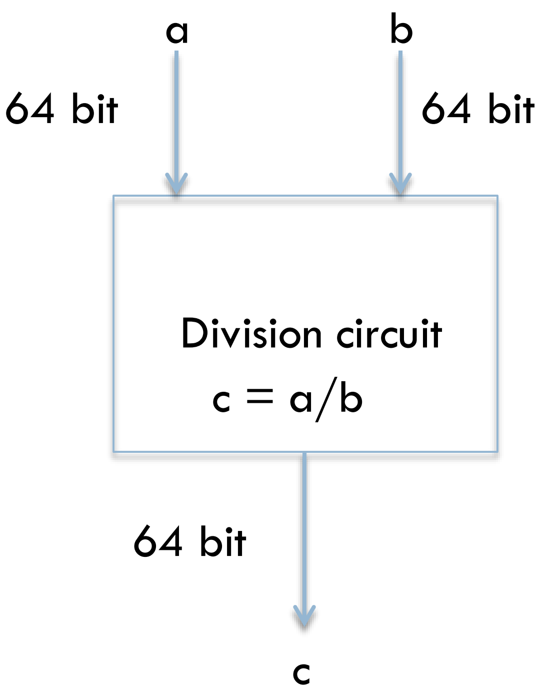
CAV 2013

Life in the 21st Century!



How do we guarantee that the systems work *<u>correctly</u>* ?

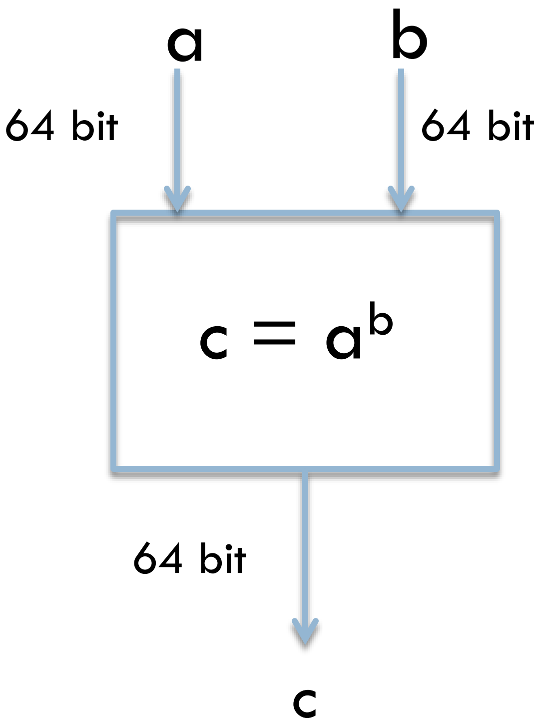
Motivating Example



How do we verify that this circuit works ?

- Formal Verification – Not Scalable!
- Randomly sample some a's and b's
  - Wait! None of the circuits in the past faulted when $10 < b < 40$
  - Finite resources!
- Lets sample from regions where it is likely to fault

Constraints Design



Designing Constraints

- Designers:
  1. $100 < b < 200$
  2. $300 < a < 451$
  3. $40 < a < 50$ and $30 < b < 40$
- Past Experience:
  1. $400 < a < 2000$
  2. $120 < b < 230$
- Users:
  1. $1000<a < 1100$
  2. $20000 < b < a < 22000$

Problem: How can we uniformly sample the values of a and b satisfying the above constraints?

Uniform Generation of SAT-Witnesses

Set of Constraints

SAT Formula

Given a SAT formula, can one uniformly sample solutions without enumerating all solutions

Uniform Generation of SAT-Witnesses

Uniform Generation of SAT-Witnesses

Set of Constraints

↓

SAT Formula

↓

Given a SAT formula, can one uniformly sample solutions without enumerating all solutions while scaling to real world problems?

Scalable Uniform Generation of SAT-Witnesses

Overview

- Prior Work & Our Approach
- Theoretical Results
- Experimental Results
- Where do we go from here?

Prior Work

BDD-based
Guarantees: strong
Performance: weak

SAT-based heuristics
Guarantees: weak
Performance: strong

INDUSTRY

Theoretical Work
Guarantees: strong
Performance: weak

Heuristic Work
Guarantees: weak
Performance: strong

ACADEMIA

BGP Algorithm

XORSample'

Our Contribution

BDD-based
Guarantees: strong
Performance: weak

SAT-based heuristics
Guarantees: weak
Performance: strong

INDUSTRY

UniWit
Guarantees : strong
Performance: strong

Theoretical Work
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ACADEMIA

BGP Algorithm

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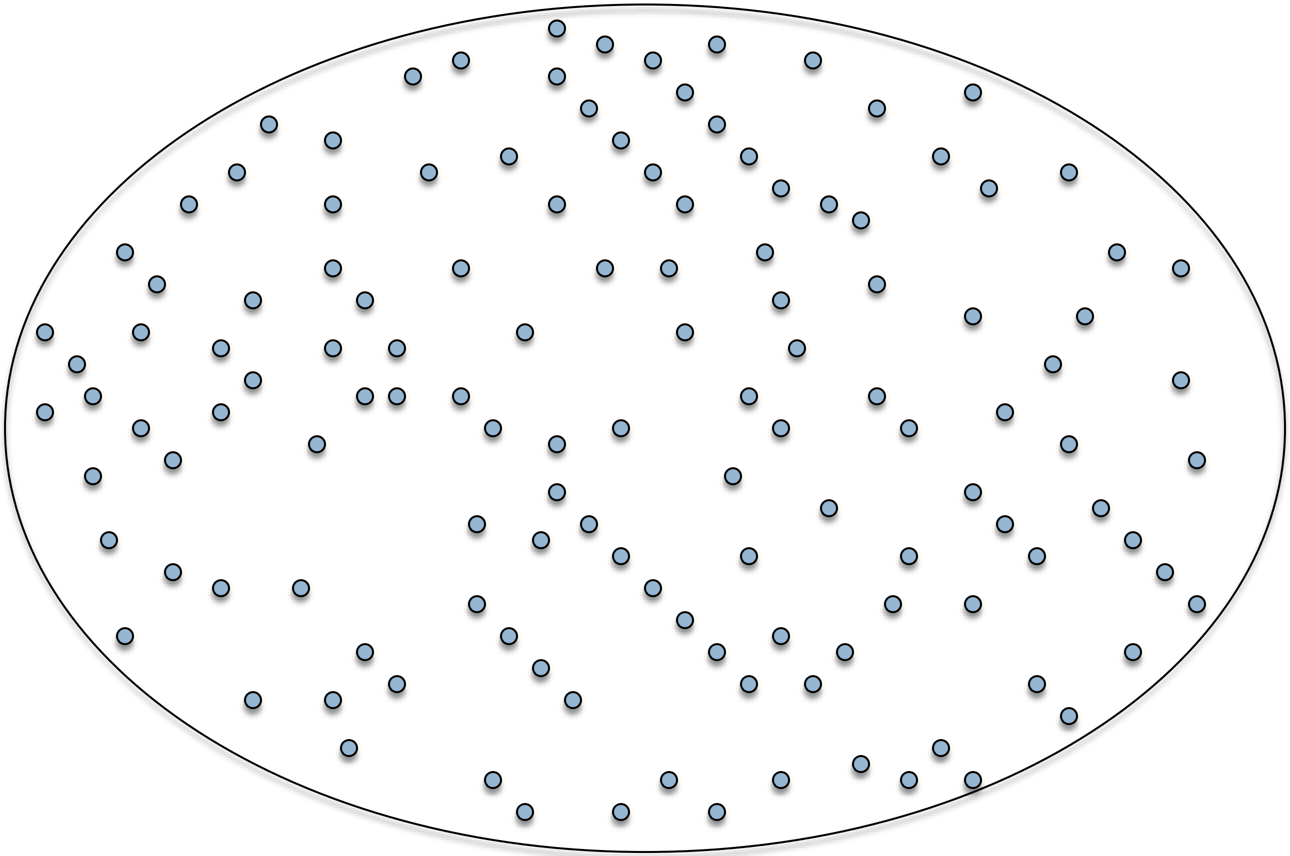
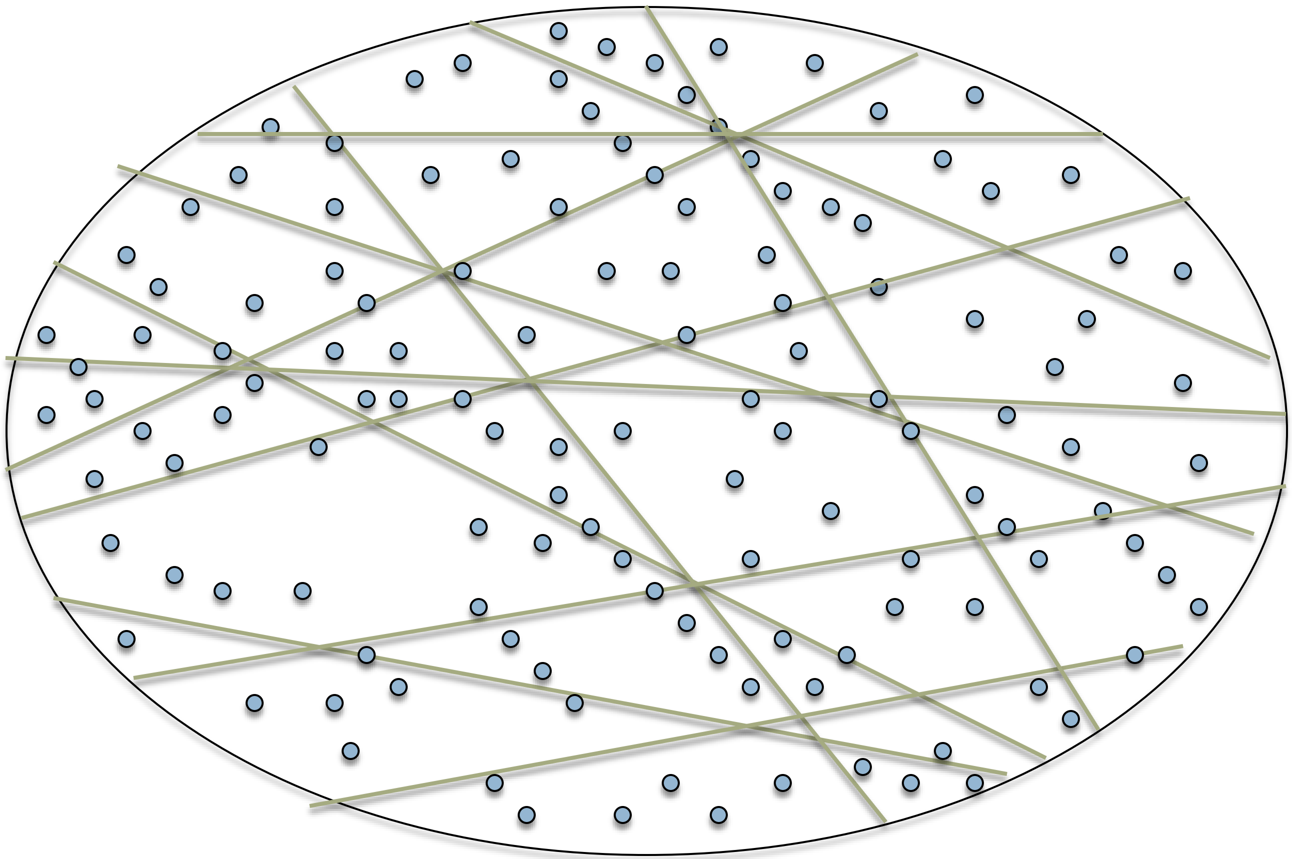
Central Idea



Partitioning into equal "small" cells



How to Partition?

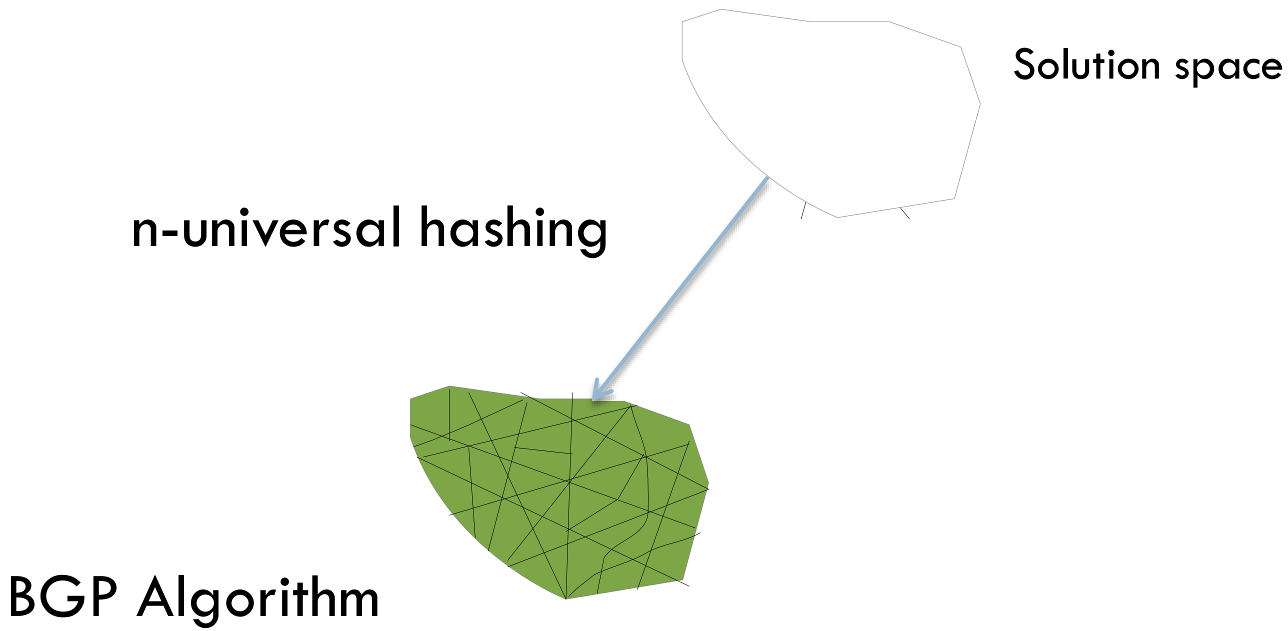
How to partition into roughly equal small cells of solutions without knowing the distribution of solutions?

Universal Hashing
[Carter-Wegman 1979, Sipser 1983]

Lower Universality ➡ Lower Complexity

- H(n,m,r): Family of r-universal hash functions mapping $\{0,1\}^n$ to $\{0,1\}^m$ (2^n elements to 2^m cells)
- Higher the r => Stronger guarantees on range of size of cells
- r-wise universality => Polynomials of degree r-1
- Lower universality => lower complexity

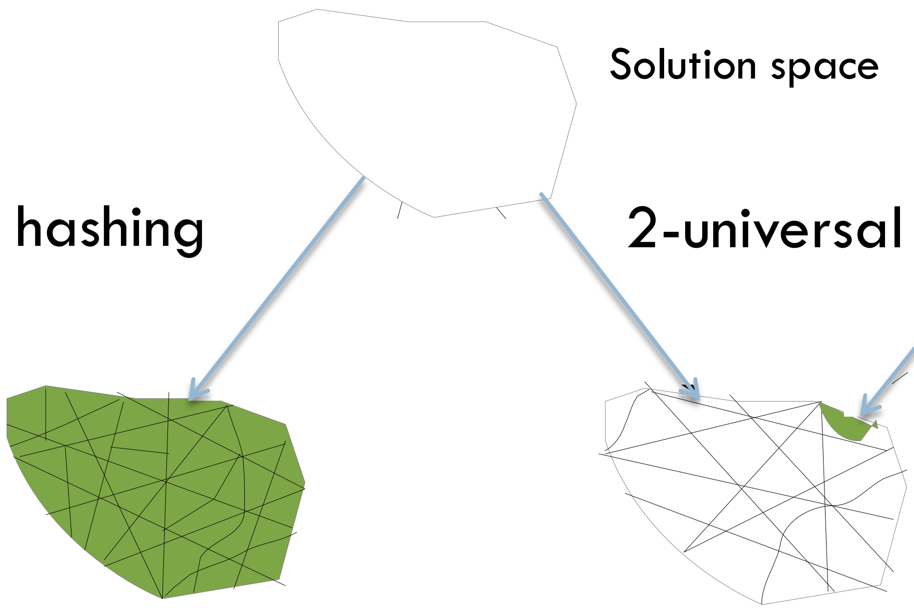
Hashing-Based Approaches



All cells should be small

Uniform Generation

Scaling to Thousands of Variables



BGP Algorithm

UniWit

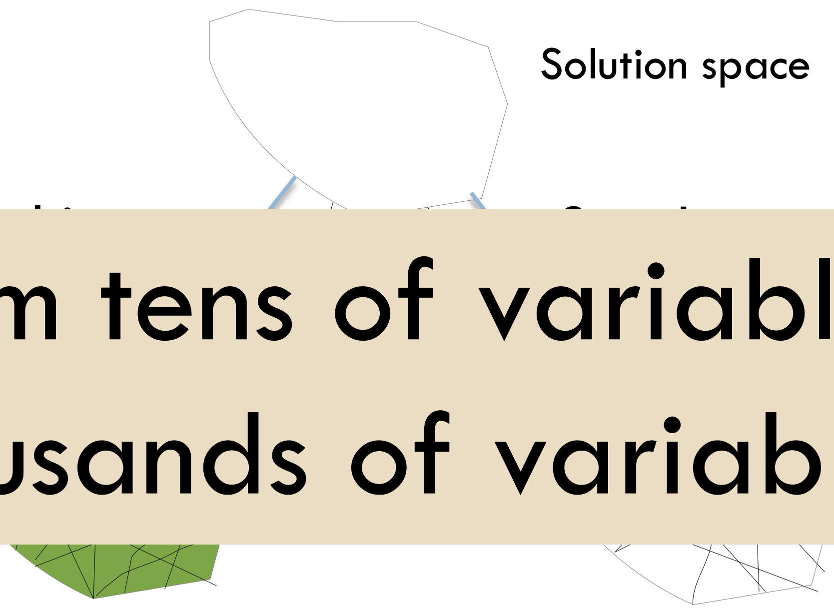
All cells should be small

Only a randomly chosen cells needs to be "small"

Uniform Generation

Near Uniform Generation

Scaling to Thousands of Variables



Solution space

n-univ

From tens of variables to thousands of variables!

BGP Algorithm

UniWit

All cells should be small

Only a randomly chosen cells needs to be “small”

Uniform Generation

Near Uniform Generation

Highlights

- Employs XOR-based hash functions instead of computationally infeasible algebraic hash functions
- Uses off-the-shelf SAT solver CryptoMiniSAT (MiniSAT+XOR support)

Strong Theoretical Guarantees

- Uniformity

For every solution y of R_F

Pr [y is output] = $1/|R_F|$

Strong Theoretical Guarantees

- Near Uniformity

For every solution y of R_F

Pr [y is output] >= $^1/8 \times 1/|R_F|$

- Success Probability

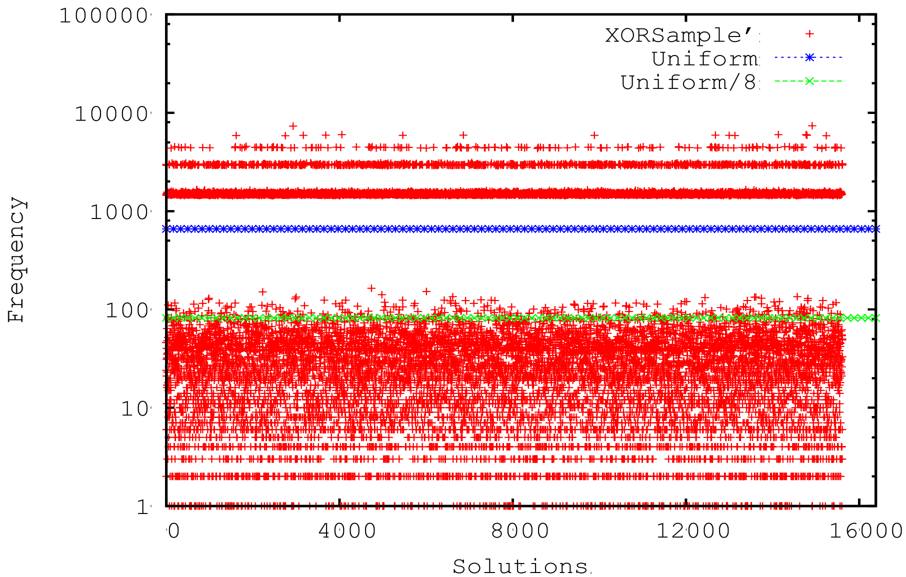
Algorithm UniWit succeeds with probability at least 1/8

- Polynomial: $O(n^{3/2})$ calls to SAT Solver

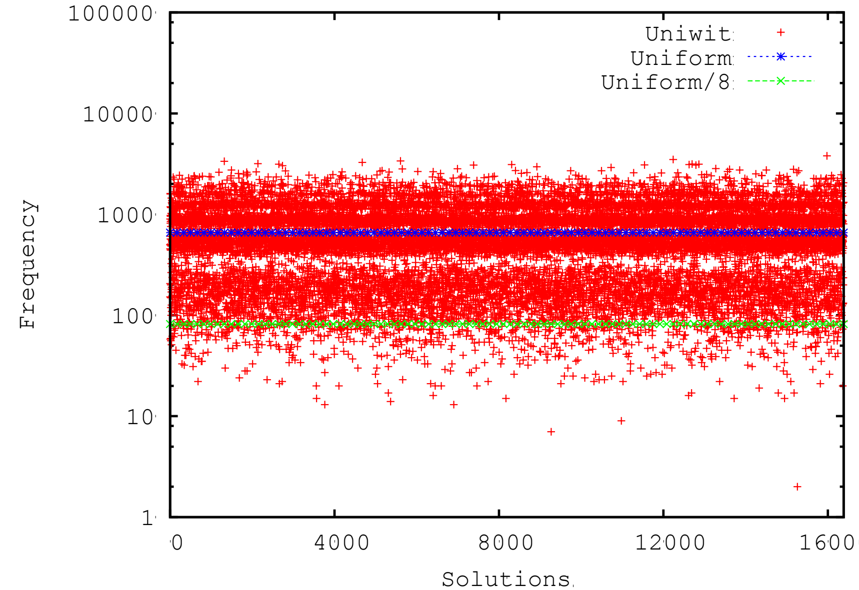
Experimental Methodology

- Benchmarks (over 200)
  - Bit-blasted versions of word level constraints from VHDL designs
  - Bit-blasted versions from SMTLib version and ISCAS'85
- Objectives
  - Comparison with algorithms **BGP** & **XORSample'**
    - **Uniformity**
    - **Performance**

Better Uniformity than State-of-art Generators



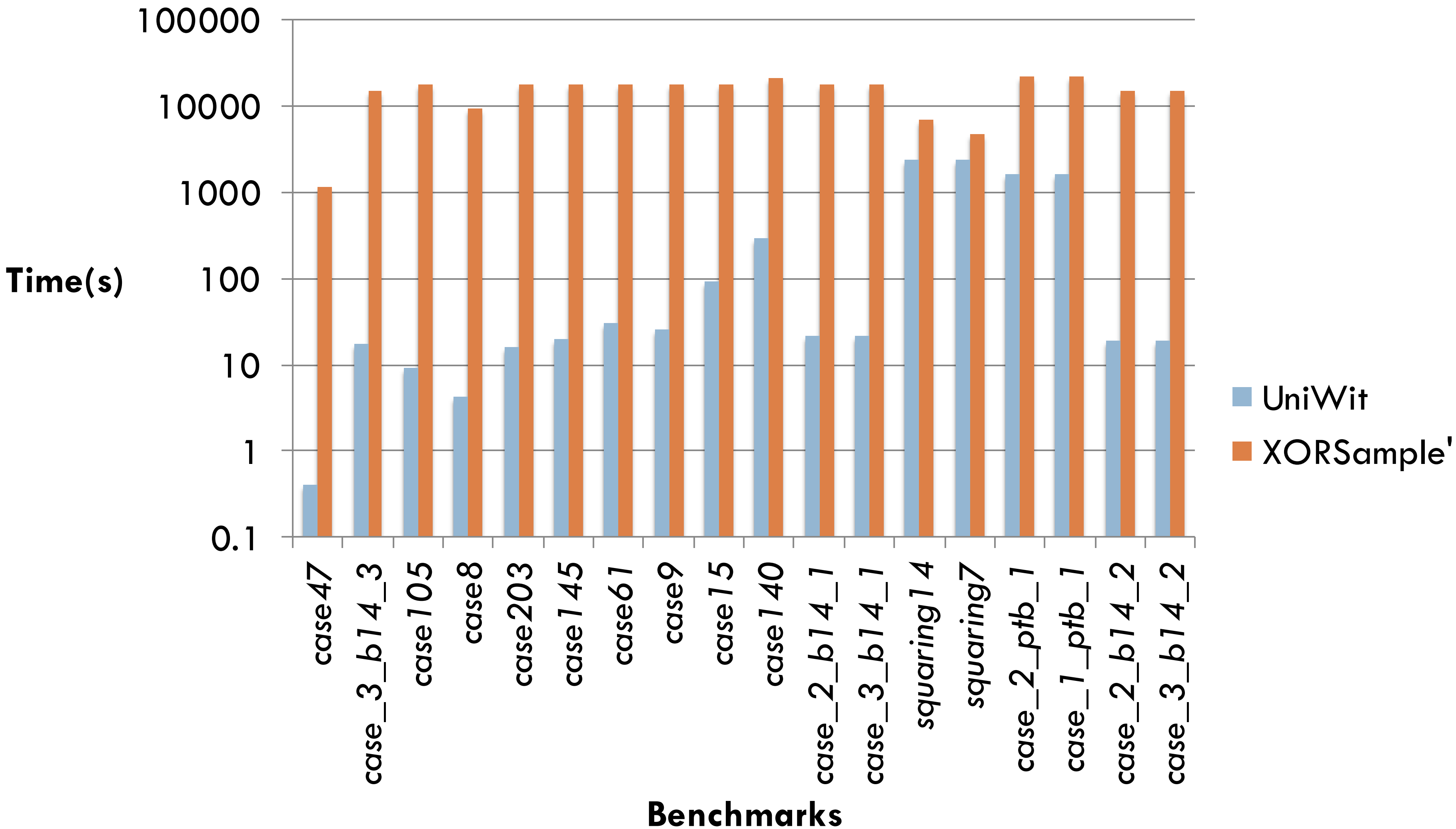
XORSample'



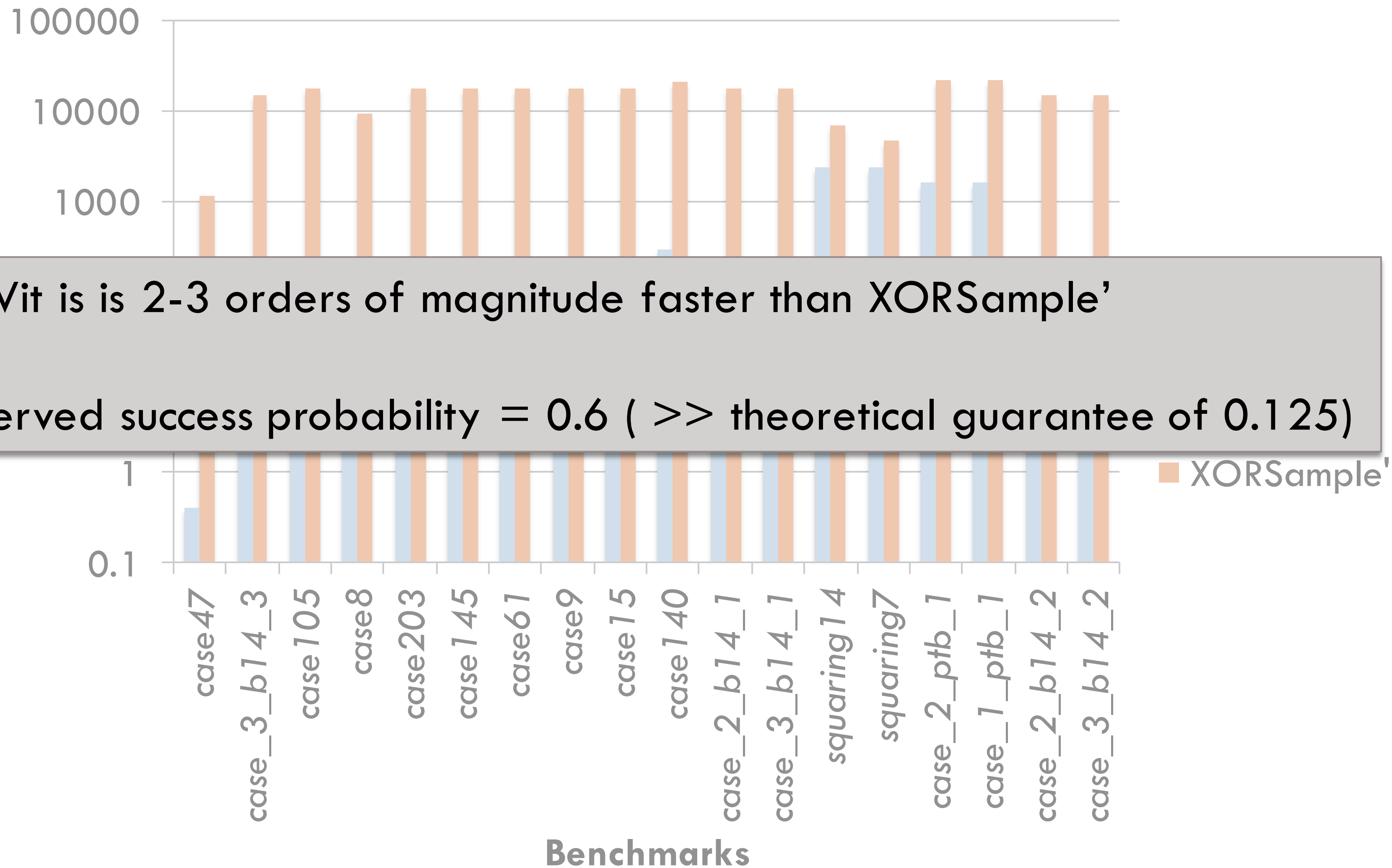
UniWit

- Benchmark: case110.cnf; #var: 287; #clauses: 1263
- Total Runs: **1.08×10^8**; Total Solutions : **16384**
- **XORSample'** could not find 772 solutions and more than 250 solutions were generated only once

2-3 Orders of Magnitude Faster



2-3 Orders of Magnitude Faster



- UniWit is is 2-3 orders of magnitude faster than XORSample'
- Observed success probability = 0.6 (>> theoretical guarantee of 0.125)

Key Takeaways

- Uniform sampling is an important problem
- Prior work didn't scale or offered weak guarantees
- We use 2-wise independent hash function to divide solution space into "small" partitions
- Only a randomly chosen partition has to be small
- Theoretical guarantees of near uniformity
- Major improvements in running time and uniformity compared to the existing generators
- Tool is available at http://www.cfdvs.iitb.ac.in/reports/UniWit/

Where Do We Go From Here?

- Extension to SMT
- Extending the technique to model counting (CP'13)
- Stronger Guarantees
- Efficient hash functions

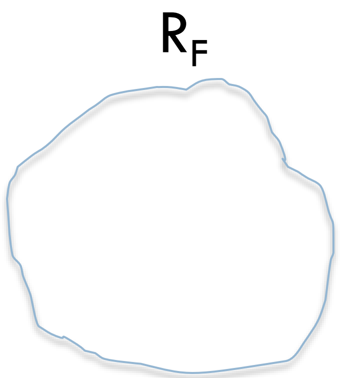
Discussion

Acknowledgments

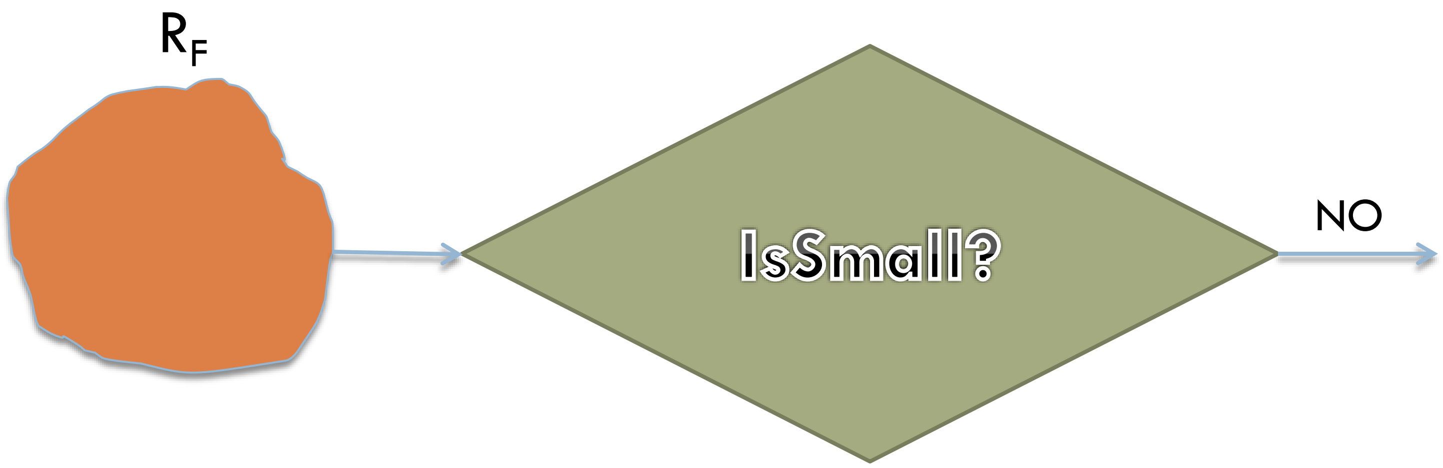
- NSF
- ExCAPE
- Intel
- BRNS, India
- Sun Microsystems
- Sigma Solutions,Inc

Thank You for your attention!

UniWit



UniWit



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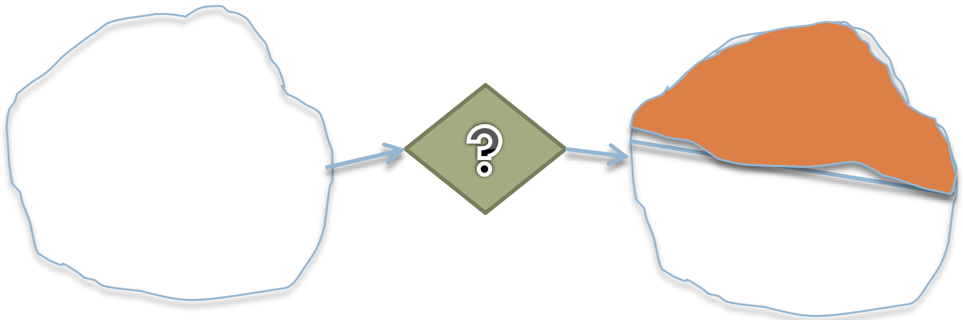

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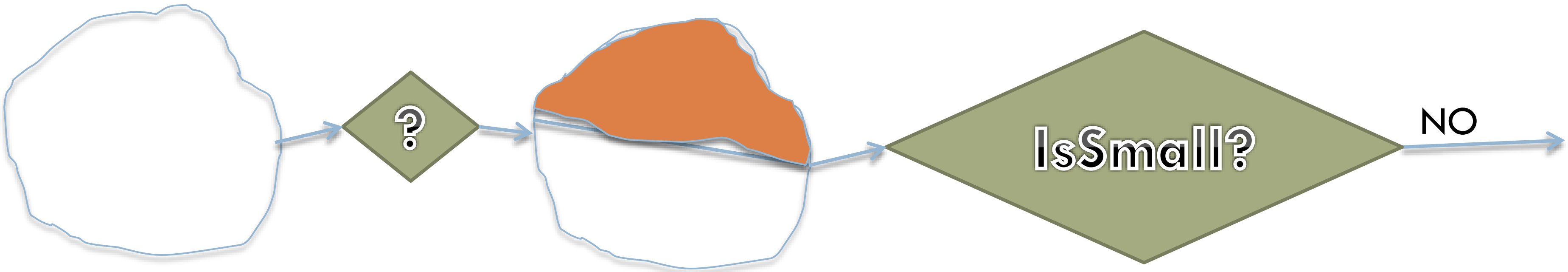

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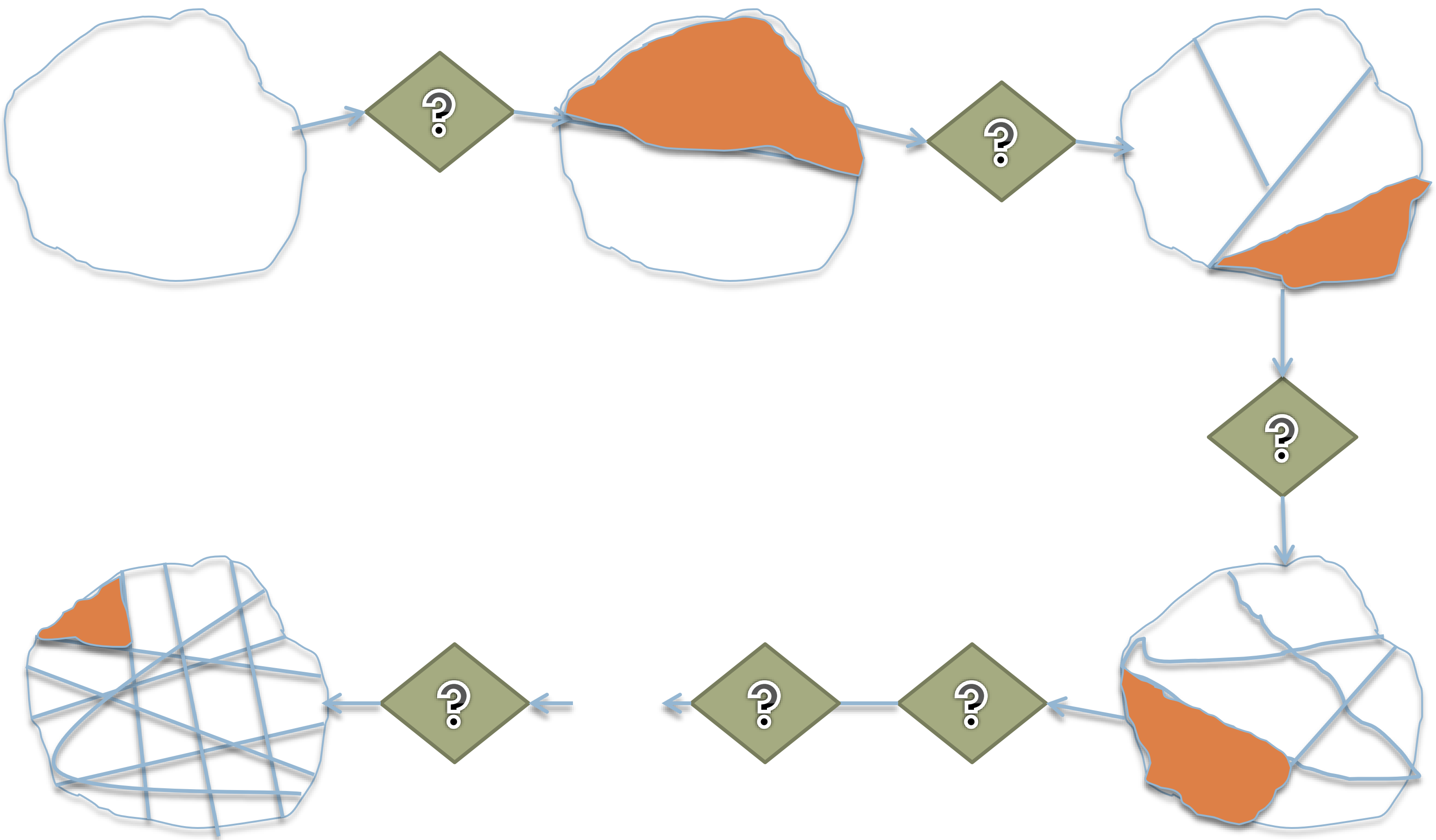

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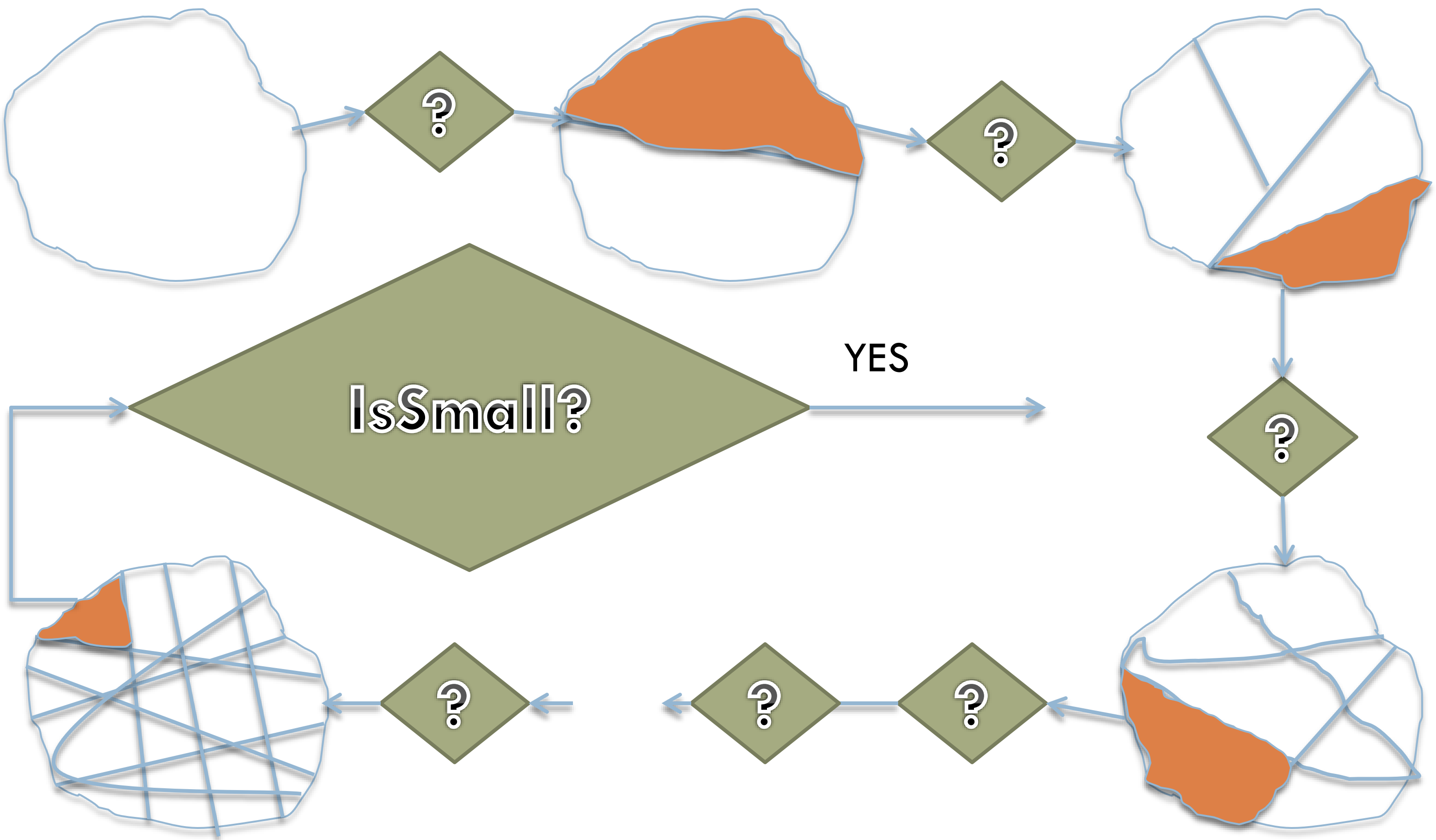

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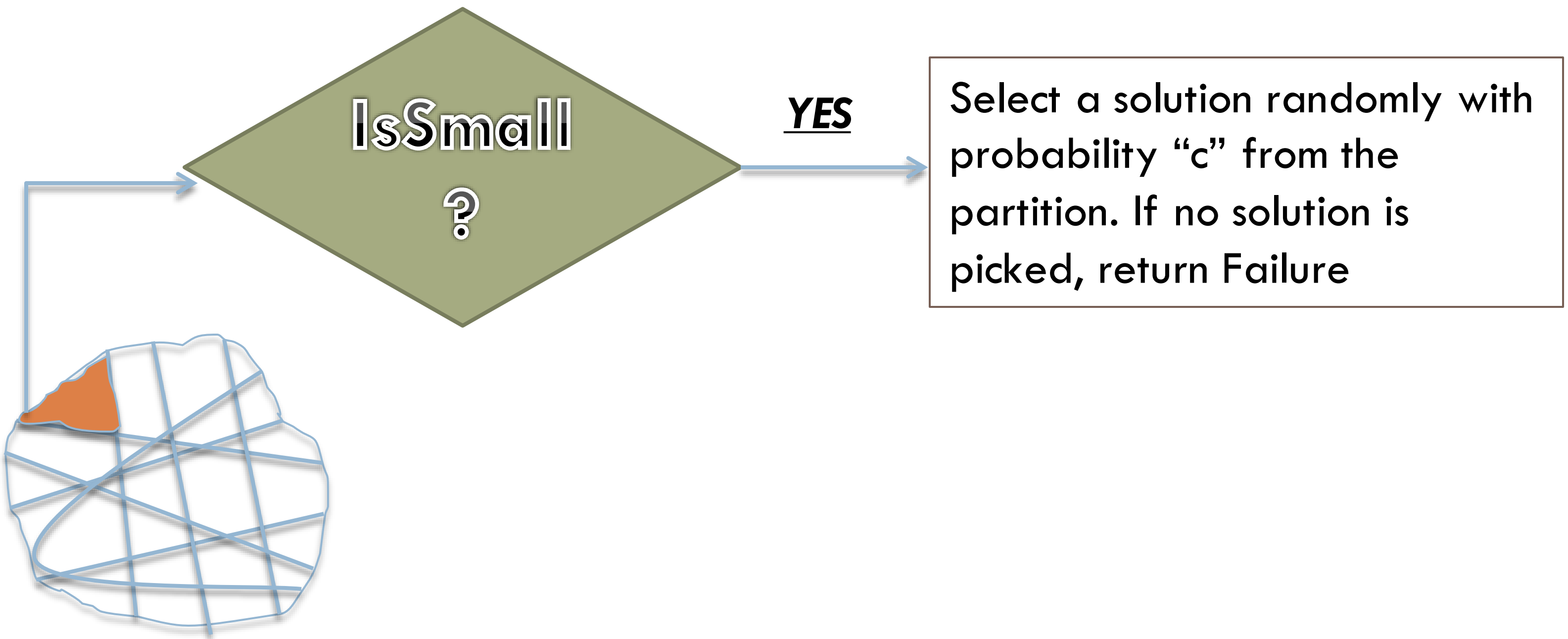
IsSmall
?
YES
Select a solution randomly with probability “c” from the partition. If no solution is picked, return Failure