A Scalable Approximate Model Counter

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What is Model Counting?

- Given a SAT formula F
- R_F: Set of all solutions of F
- Problem (#SAT): Estimate the number of solutions of F (#F) i.e., what is the cardinality of R_F?
- E.g., F = (a v b)
- R_F = {(0,1), (1,0), (1,1)}
- The number of solutions (#F) = 3

#P: The class of counting problems for decision problems in NP!

Practical Applications

Exciting range of applications!

- Probabilistic reasoning/Bayesian inference
- Planning with uncertainty
- Multi-agent/ adversarial reasoning

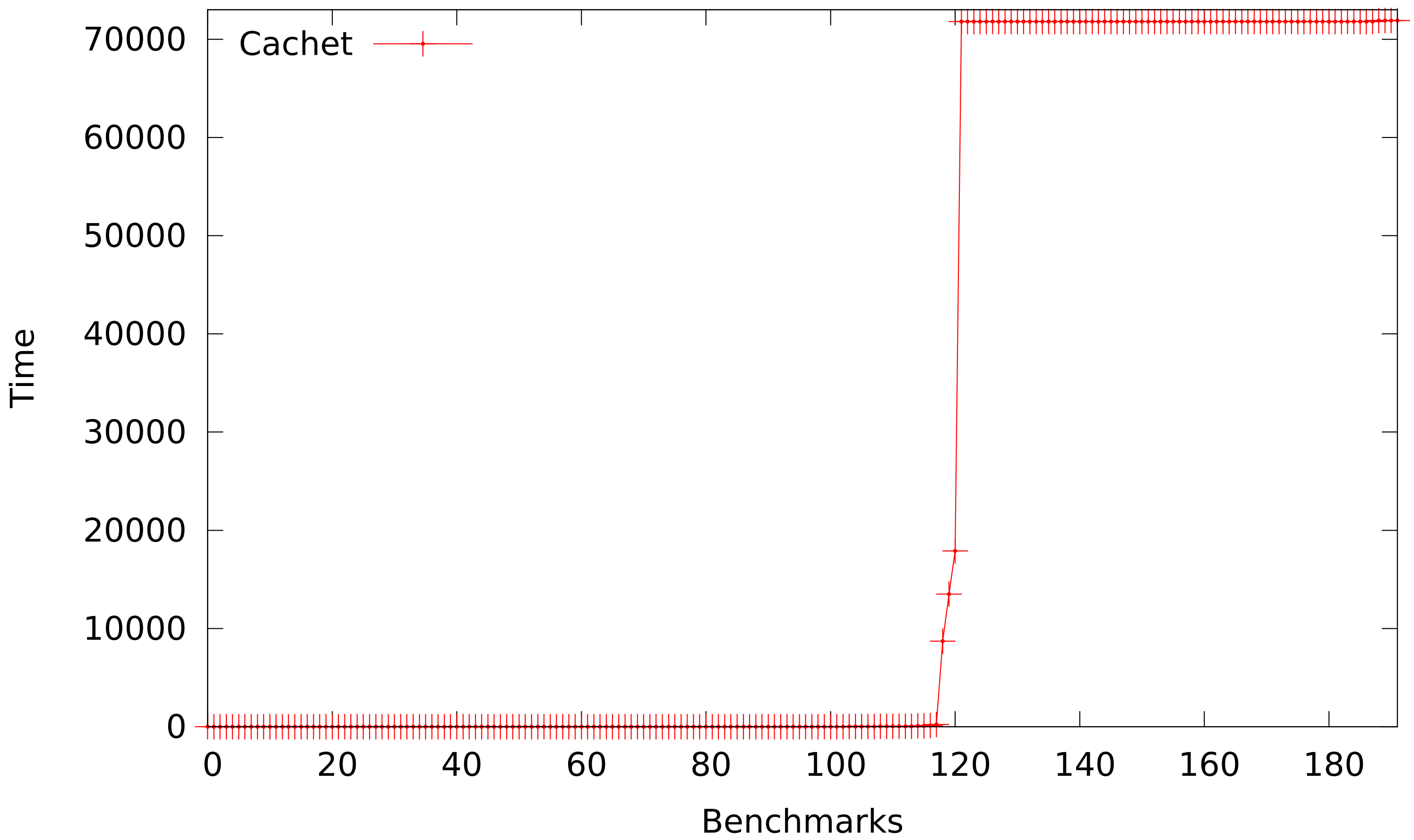
[Roth 96, Sang 04, Bacchus 04, Domshlak 07]

But it is hard!

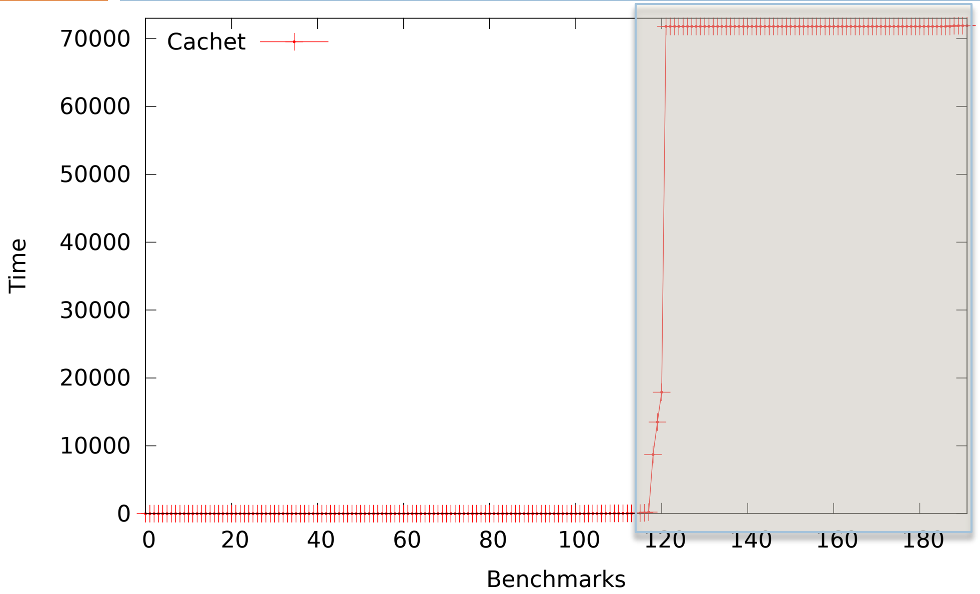
- #SAT is #P-complete
  - Even for counting solutions of 2-CNF SAT

- #P is really hard!
  - Believed to be much harder than NP-complete problems
  - $\mathrm{PH} \subseteq \mathrm{P}^{\#\mathrm{P}}$

The Hardness of Model Counting



The Hardness of Model Counting



Can we do better?

Approximate counting (with guarantees) suffices for most of the applications

Prior Work

Input Formula: F; Total Solutions: #F; Return Value: C

Counters	Guarantee	Confidence	Remarks
Exact counter (e.g. sharpSAT, Cachet)	$C = \#F$	1	Poor Scalability
Lower bound counters (e.g. MBound, SampleCount)	$C \leq \#F$	δ	Very weak guarantees
Upper bound counters(e.g. MiniCount)	$C \geq \#F$	δ	Very weak guarantees

Approximate Model Counting

Design an approximate model counter G:

- inputs:
  - CNF formula F
  - tolerance ε
  - confidence δ

- the count returned by it is within ε of the #F with confidence at least δ

Approximate Model Counting

Approximate Model Counting

Design an approximate model counter G:

- inputs:
  - CNF formula F
  - tolerance ε
  - confidence δ

- the count returned by it is within ε of the #F with confidence at least δ and scales to real world problems

Scalable Approximate Model Counting

Lies in the 2nd level of Polynomial hierarchy: Σ_2^P

Our Contribution

Input Formula: F; Total Solutions: #F

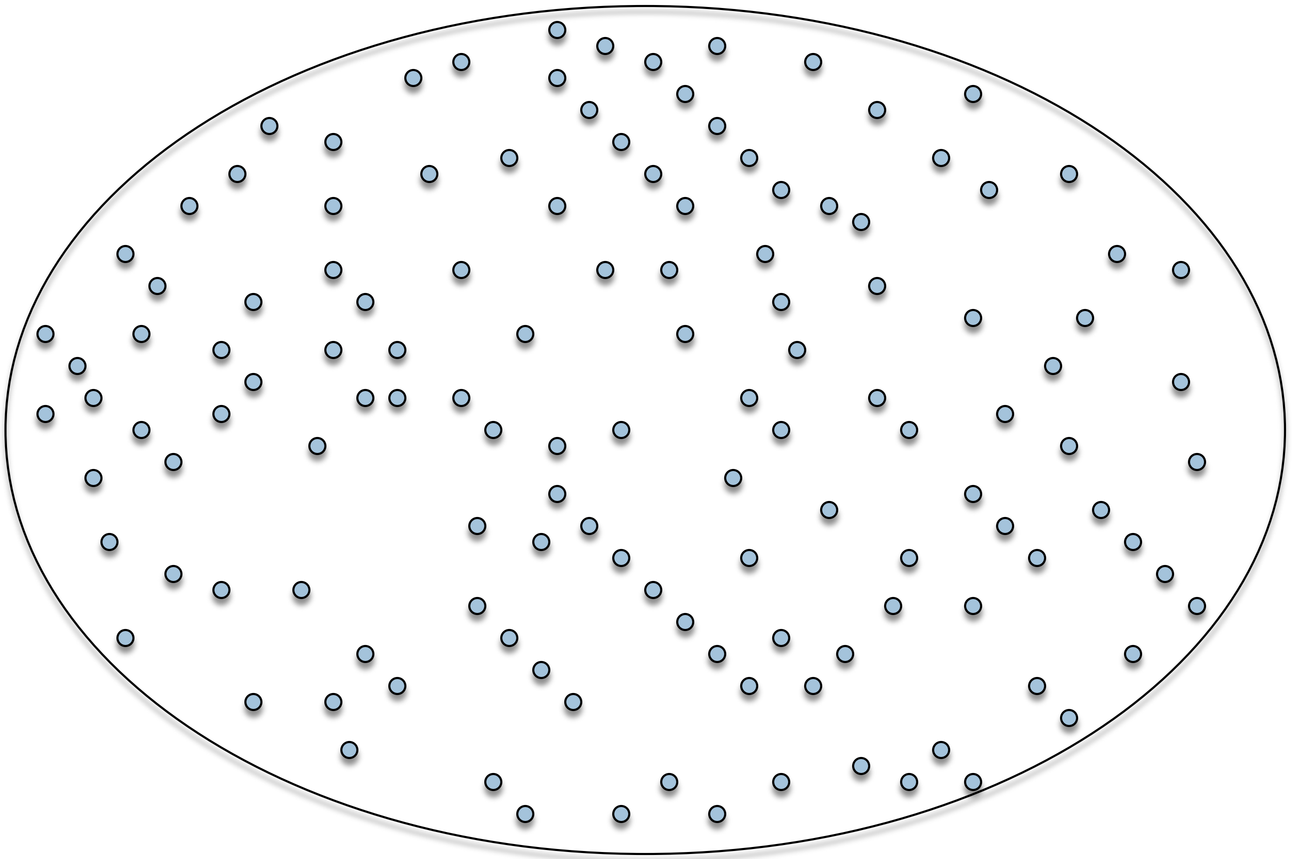
Counters	Guarantee	Confidence	Remarks
Exact counter (e.g. sharpSAT, Cachet)	C = #F	1	Poor Scalability
ApproxMC	$\#F/(1+\varepsilon) \leq C \leq (1+\varepsilon)\ \#F$	δ	Scalability + Strong guarantees

The First Scalable Approximate Model Counter

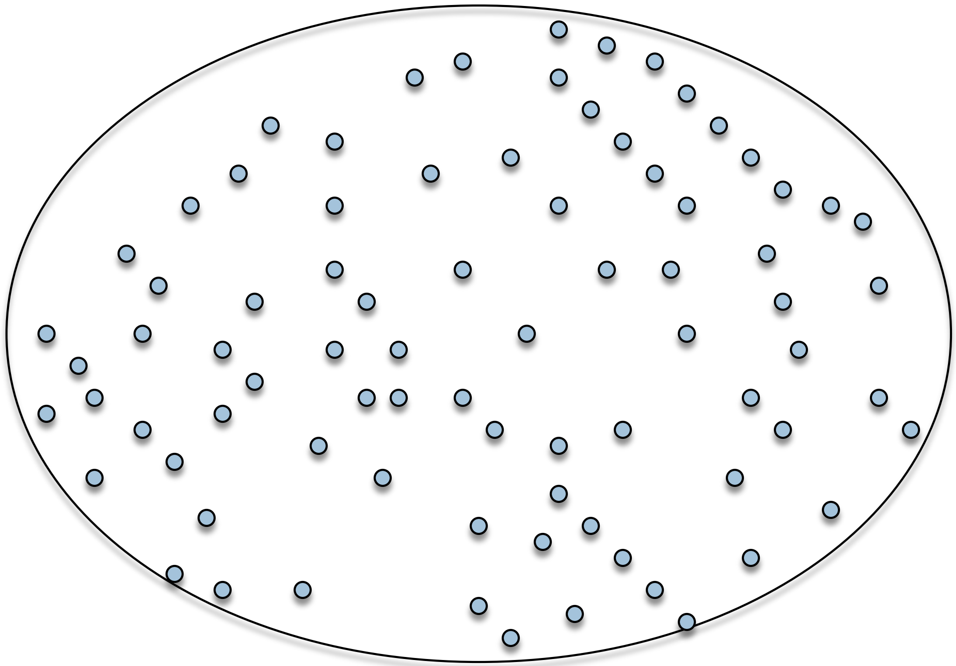
Overview

- Our approach
- Theoretical results
- Experimental results
- Where do we go from here?

How do we count?



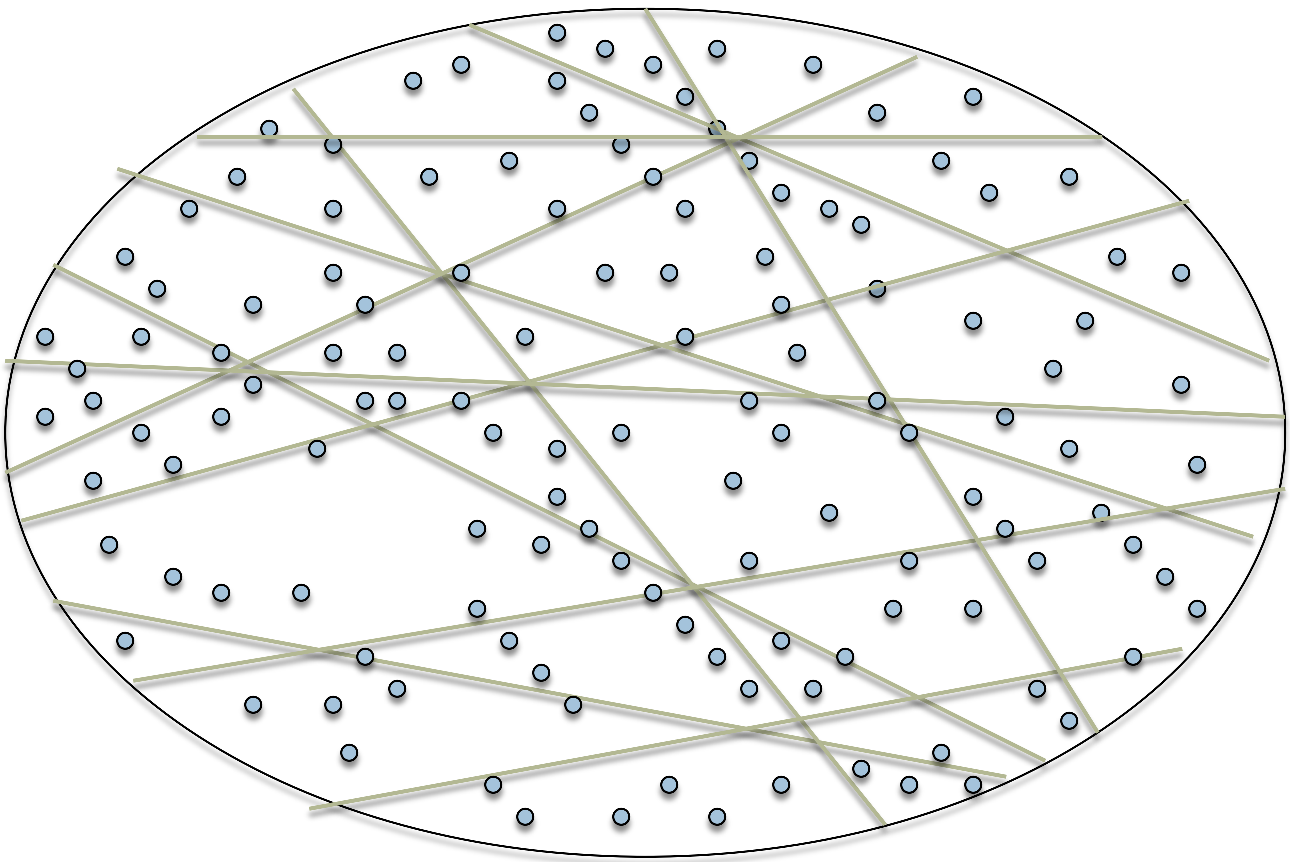
Explicit Enumeration: Not Scalable



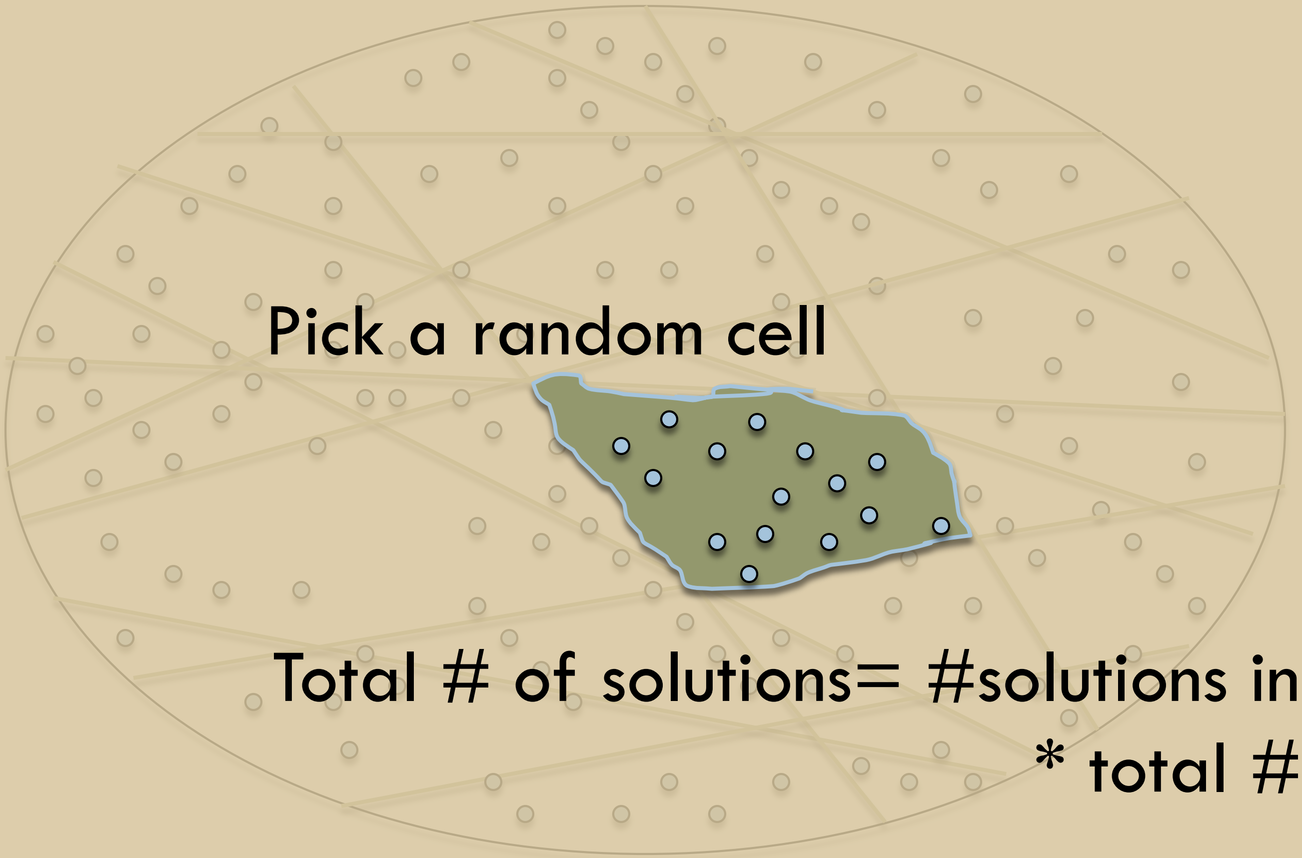
- Enumerate (almost) all solutions
- Exact Counting!
- Cachet, Relsat, sharpSAT

Not Scalable!

Counting through Partitioning



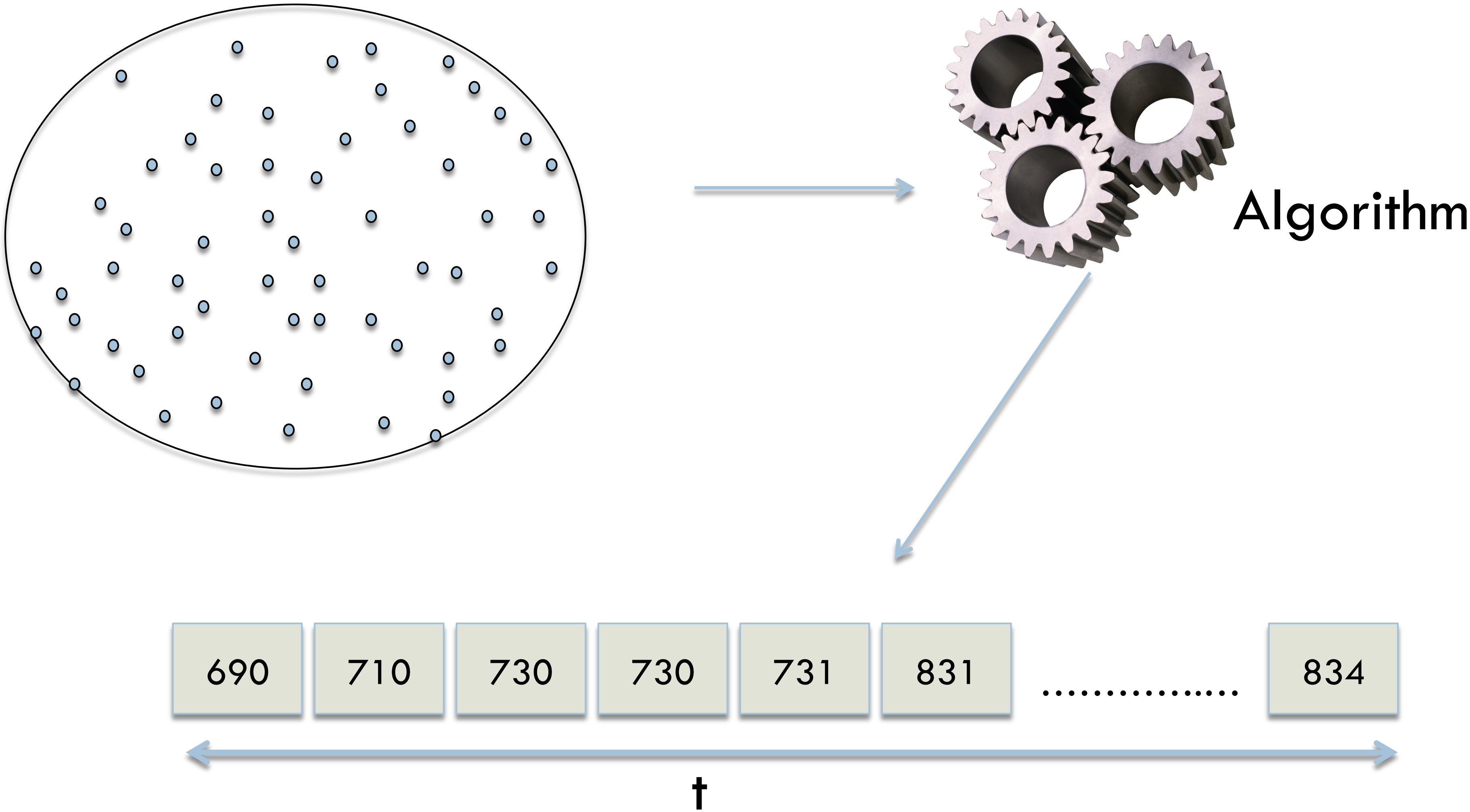
Counting through Partitioning



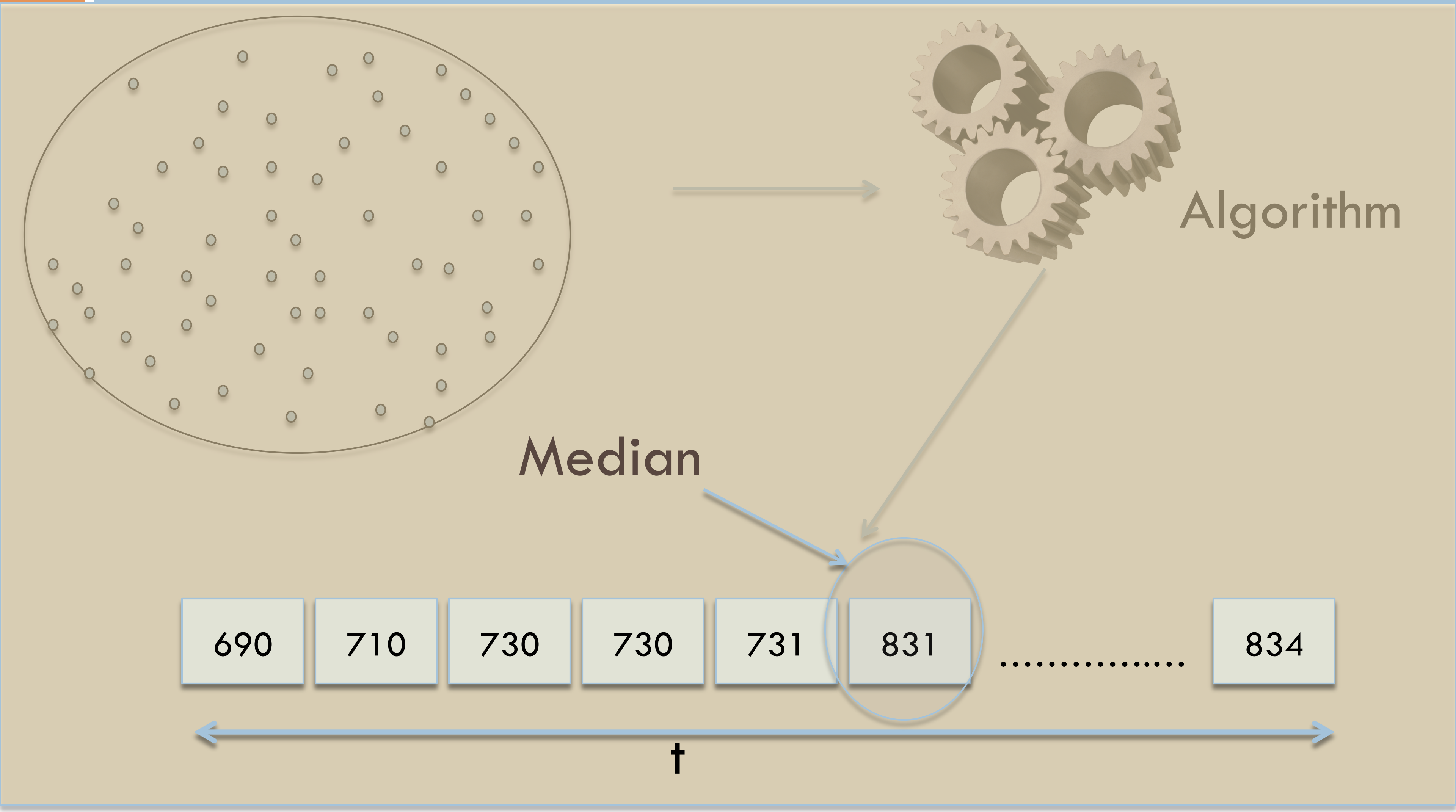
Pick a random cell

Total # of solutions= #solutions in the cell
* total # of cells

Algorithm in Action



Algorithm in Action



How to Partition?

How to partition into roughly equal small cells of solutions without knowing the distribution of solutions?

Universal Hashing
[Carter-Wegman 1979, Sipser 1983]

Universal Hashing

- Hash functions from mapping $\{0,1\}^n$ to $\{0,1\}^m$ (2^n elements to 2^m cells)

- Random inputs => All cells are *roughly* small

- Universal hash functions:
  - Adversarial (any distribution) inputs => All cells are *roughly* small

- Need stronger bounds on range of the size of cells

Higher Universality ➡ Stronger Guarantees

- H(n,m,r): Family of r-universal hash functions mapping $\{0,1\}^n$ to $\{0,1\}^m$ (2^n elements to 2^m cells)

- Higher the r => Stronger guarantees on range of size of cells

- r-wise universality => Polynomials of degree r-1

- Lower universality => lower complexity

Highlights of Our Hashing

- Employs XOR-based hash functions instead of computationally infeasible algebraic hash functions

- Uses off-the-shelf SAT solver CryptoMiniSAT (MiniSAT+XOR support)

Strong Theoretical Results

ApproxMC (CNF: F, tolerance: ε, confidence:δ)

Suppose ApproxMC(F,ε,δ) returns C. Then,

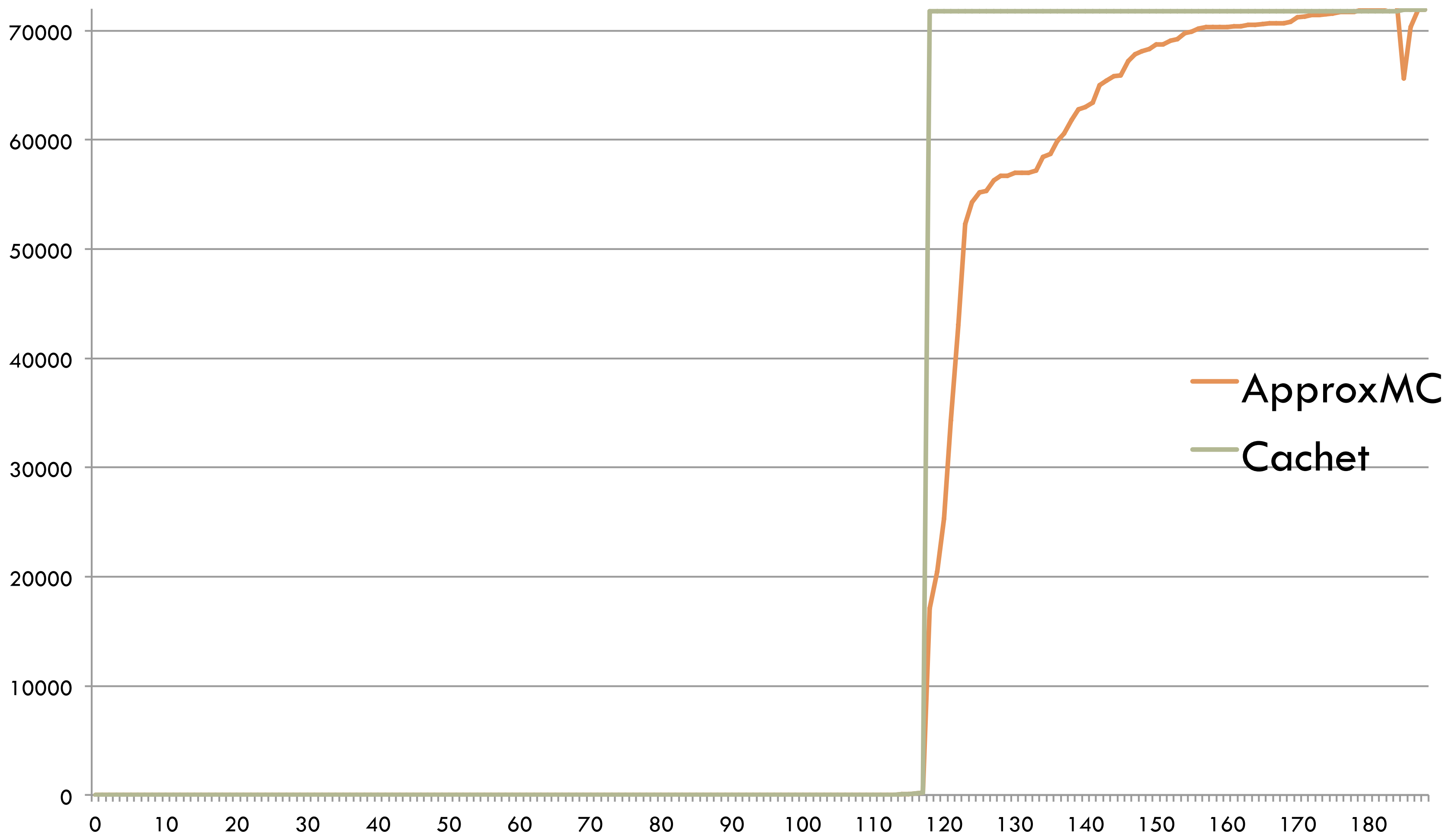
$$\mathbf{Pr}\left[\ \#\mathbf{F}/(1+\varepsilon) \leq \mathbf{C} \ \leq (1+\varepsilon)\ \#\mathbf{F}\ \right] \geq \delta$$

ApproxMC runs in time polynomial in log $(1\text{-}\delta)^{-1}$, $|F|$, ε^{-1} relative to SAT oracle

Experimental Methodology

- Benchmarks (over 200)
  - Grid networks, DQMR networks, Bayesian networks
  - Plan recognition, logistics problems
  - Circuit synthesis
- Tolerance: $\varepsilon = 0.75$, Confidence: $\delta = 0.9$
- Objectives
  - Comparison with exact counters (Cachet) & bounding counters (MiniCount, Hybrid-MBound, SampleCount)
    - Performance
    - Quality of bounds

Results: Performance Comparison



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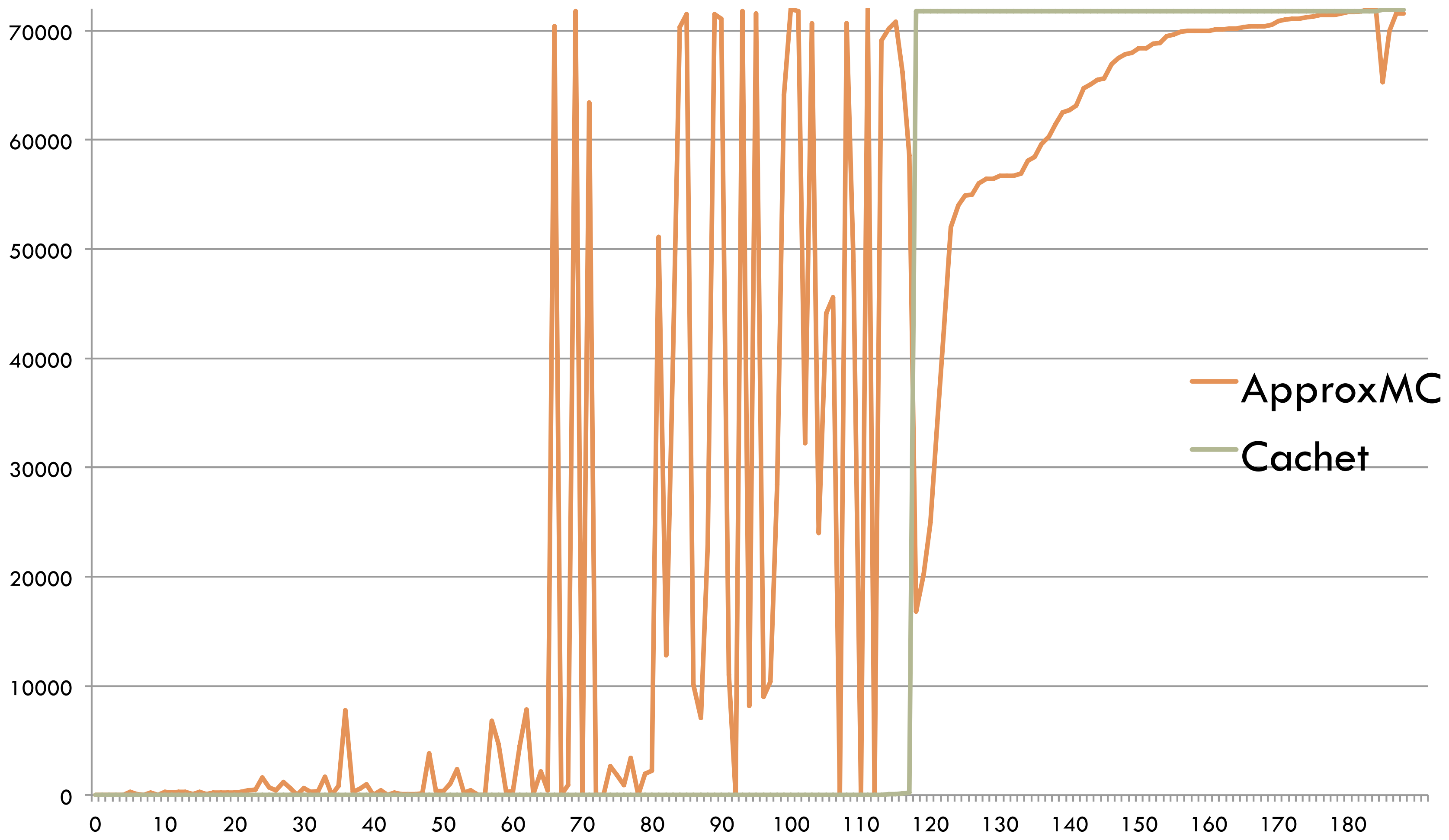
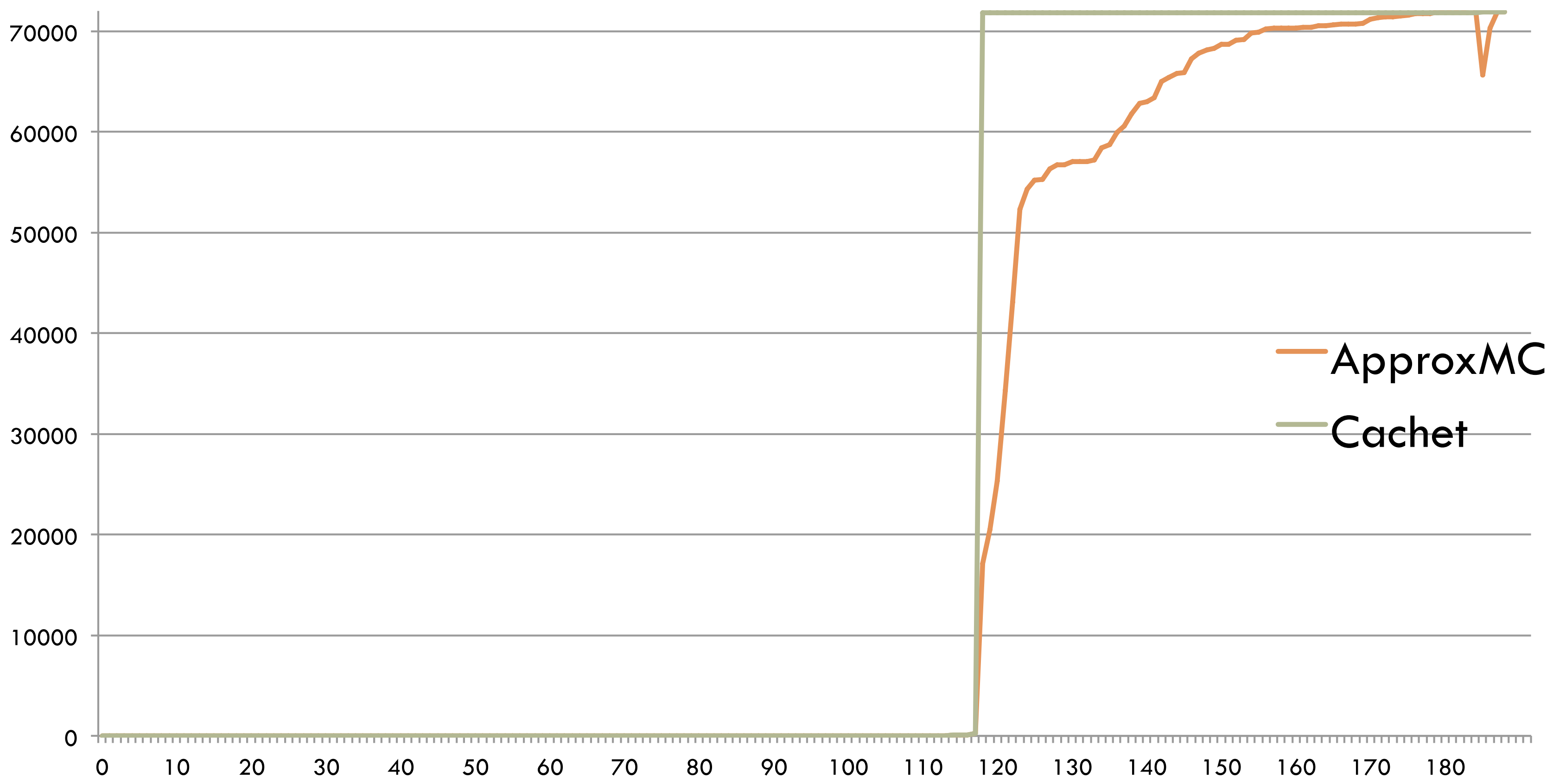
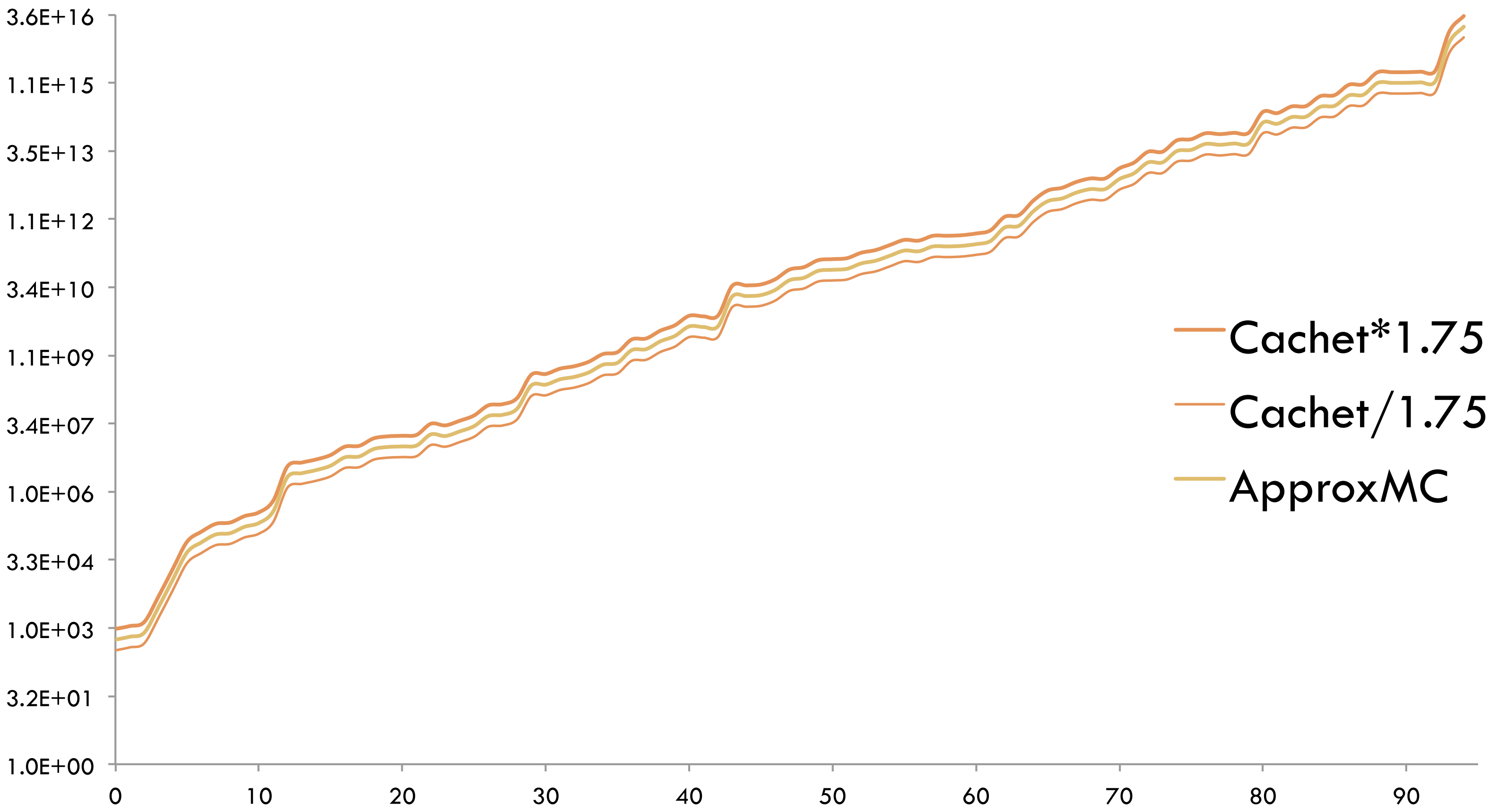


Can Solve a Large Class of Problems



Large class of problems that lie beyond the exact counters but can be computed by ApproxMC

Mean Error: Only 4% (allowed: 75%)



Mean error: 4% – much smaller than the theoretical guarantee of 75%

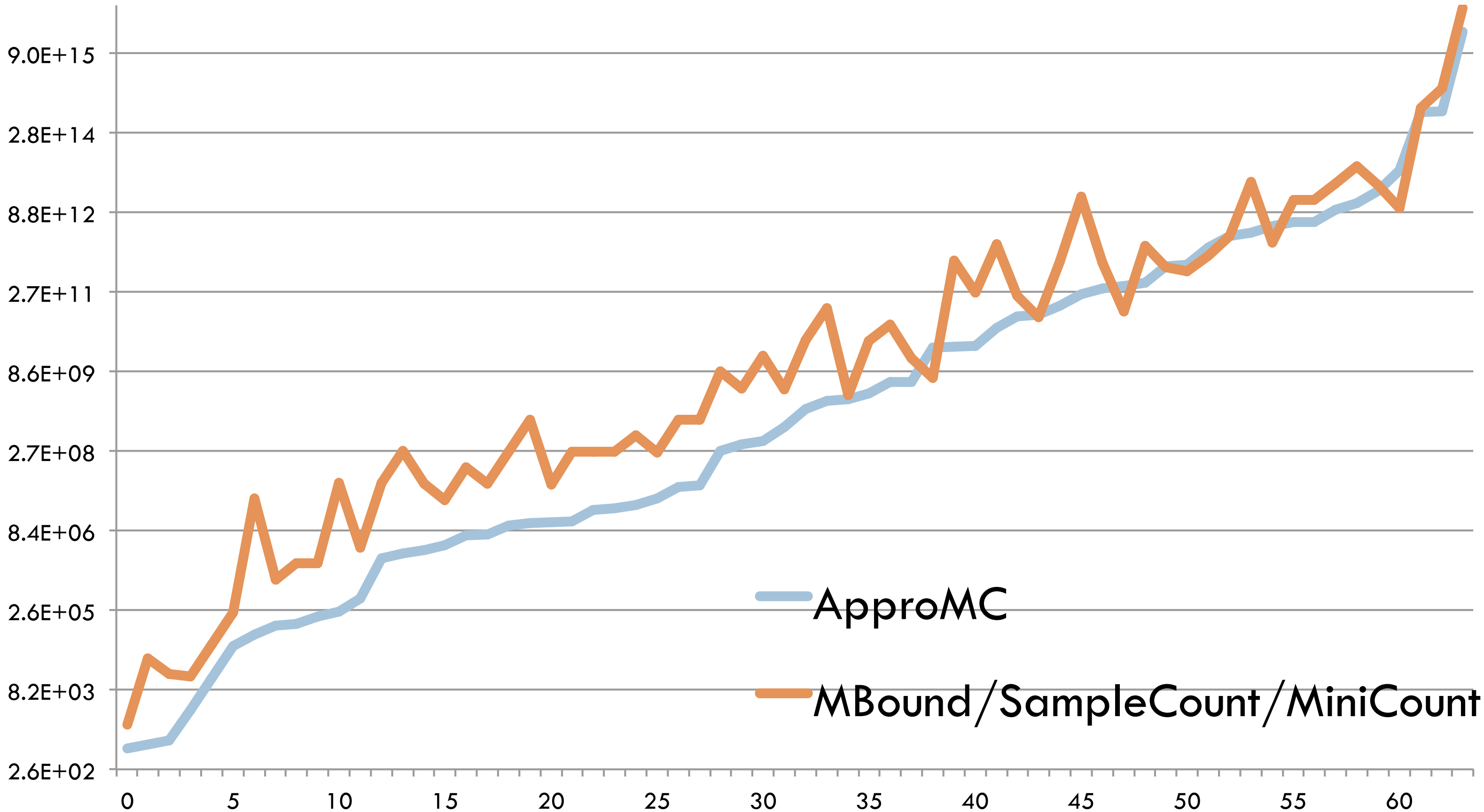
Results: Bounding Counters

- Range of count from bounding counters = C_2-C_1
  - C_1: From lower bound counters(MBound/SampleSAT)
  - C_2: From upper bound counters (MiniCount)

- Range from ApproxMC: $[C/(1+\varepsilon), (1+\varepsilon)C]$

- Smaller the range, better the algorithm!

Better Bounds Than Existing Counters



ApproxMC improved the upper bounds significantly while also improving the lower bounds

Key Takeaways

- Many practical applications can be reduced to (approximate) model counting
- ApproxMC is the first scalable approximate model counter
- Uses easy-to-implement linear hash functions
- Major improvements in performance and quality of bounds compared to existing counters.
- Tools is available at http://www.cs.rice.edu/~kgm2/ModelCounting/

Where do we go from here?

- Ongoing work : Probabilistic Inference

- Further scaling: Efficient hash functions

- Extension to CSP and SMT domains

Discussion

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Thank You for your attention!